Food Waste in Omnichannel Grocery Retail: The Role of Buy Online and Pick Up In Store

Jieqi Di
H. Milton Stewart School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, GA 30332, jdi7@gatech.edu

Yuwei Zhou
Kelley School of Business, Indiana University, Bloomington, IN 47405, yuwzhou@iu.edu

Fei Gao
Kelley School of Business, Indiana University, Bloomington, IN 47405, fg1@iu.edu

Food waste in grocery retail arises both on store shelves and in consumers' homes, yet the growing adoption of buy-online-pick-up-in-store (BOPS) can change how waste is generated across these two locations. We study how BOPS reshapes the effectiveness of two widely used waste-reduction interventions: grocers' investments in freshness technology and governments' per-unit waste taxes. We develop an analytical model comparing a store-only retailer with an omnichannel retailer that offers BOPS when profitable. In-store customers observe freshness before purchasing, whereas BOPS customers commit before fulfillment. Our main results are as follows. The profit and waste implications of freshness technology depend jointly on channel structure and the convenience of in-store shopping. When in-store shopping is convenient, the store-only retailer realizes a larger profit gain and reduces store waste, whereas the omnichannel retailer realizes a smaller profit gain and may experience increased store waste. When in-store shopping is inconvenient, both comparisons can reverse. A retail waste tax also creates a trade-off: it reduces store waste but increases household waste, potentially raising total waste when household spoilage is sufficiently high. Whether the BOPS option mitigates or amplifies this unintended effect depends on online convenience. When the online channel is relatively convenient, the tax-induced increase in total waste is no greater for the omnichannel retailer than for the store-only retailer; when the online channel is less convenient, this comparison can reverse. These findings show that waste interventions must be evaluated alongside retailers' channel structures and their effects on both store and household waste.

Key words: Omnichannel operations, Food waste, Consumer Behavior, Game Theory

1. Introduction

Food waste is a problem of staggering scale and considerable economic consequence in the grocery industry, where it arises simultaneously as spoilage on the store shelf and as discarded food in the consumer's home. The aggregate scale is large: globally, approximately 1.05 billion metric tons of food were wasted in 2022, roughly one-fifth of all food available to consumers (UNEP 2024), and in the U.S. an estimated 30 to 40 percent of the food supply goes uneaten (USDA 2026). Grocery

retail sits at the center of this problem, and the costs fall on both grocers and their customers simultaneously, with neither side negligible. On the retail side, unsold food at U.S. grocery stores represented a loss of $26.9 billion in 2024 alone (ReFED and WWF 2025). On the consumer side, the burden is even larger and falls heavily on groceries brought home from the store: U.S. households discard a substantial share of the food they purchase, at an estimated annual cost of $2,913 for a household of four—roughly $56 per week (EPA 2025).

Confronted with waste of this magnitude, both grocers and policymakers have begun to attack the problem directly, each through its own set of levers. Grocers act on the waste they can see on their own shelves, while governments act through regulation that reshapes the grocer's incentives.

On the retail side, grocers increasingly invest in operational technologies that slow the rate at which products deteriorate on the shelf. The need is substantial: the USDA Economic Research Service estimates average supermarket shrink rates of 12.6 percent for fresh fruits and 11.6 percent for fresh vegetables across U.S. supermarkets based on 2011–2012 data (Buzby et al. 2016). In response, grocers are deploying a growing array of freshness management technologies—real-time temperature and humidity controls, smart packaging with embedded spoilage sensors, and AI-driven shelf monitoring systems—that can meaningfully reduce the rate of in-store deterioration (Mani et al. 2025, Palanisamy et al. 2025). The potential payoff can be large: even a single additional day of shelf life can reduce waste by as much as 43.1 percent while simultaneously improving on-shelf product availability by 3.4 percent (Broekmeulen and van Donselaar 2019).

Governments have increasingly sought to address retail food waste through regulation. One approach is to impose direct disposal charges: commercial generators such as grocery retailers face per-ton landfill taxes and surcharges, including the United Kingdom's Landfill Tax and statewide disposal surcharges in Pennsylvania, Iowa, and Tennessee (HM Revenue & Customs 2024, Jones 2021, Waste Advantage Magazine 2025). Governments have also considered substantially increasing these charges; for example, a 2024 Michigan proposal would raise the state's solid-waste surcharge more than tenfold to encourage waste reduction (Brown 2024). Another approach is to mandate the separation and processing of commercial food waste. For example, Washington, D.C. and New York City require covered grocery retailers to separately collect and process organic waste, while Seattle requires commercial food-waste generators to arrange for composting or other approved processing (Council of the District of Columbia 2021, New York City Department of Sanitation 2025, Seattle Public Utilities 2026). Although these policies differ in design, they share a common economic feature: they make food waste more costly for retailers to handle or dispose of, thereby creating a financial incentive to reduce retail food waste. We refer broadly to such cost-based policy interventions as retail waste taxes.

A defining feature of the modern grocery landscape is the rapid shift toward omnichannel shopping, and in particular the rise of buy-online-pick-up-in-store (BOPS). More than 90 percent of U.S. grocery shoppers now shop across both online and in-store channels, and online grocery shopping has grown substantially in recent years, with digitally engaged shoppers spending significantly more than their in-store-only counterparts (Food Industry Association and NielsenIQ 2025). BOPS has become one of the most prominent manifestations of this shift: customers place their orders online and collect their purchases at the physical store. Unlike home delivery, BOPS keeps the customer in direct contact with the retail environment at the point of pickup, yet—because the order is placed online—the purchase is committed before the customer can physically inspect the product. Moreover, BOPS is typically operated by the grocer itself, with orders picked and fulfilled from the grocer's own store. This combination of online commitment and in-store fulfillment has made BOPS especially attractive for grocery, where it now sits alongside in-store shopping as a standard way for households to buy perishable goods.

The two waste-reduction efforts described above—grocer freshness technology and a government waste tax—share a common premise: both target waste where it is visible, on the retail shelf. The rise of BOPS complicates that premise. Because BOPS customers commit to a purchase online without inspecting the product, they receive whatever freshness state is available at the time of fulfillment, whereas in-store customers can observe freshness and decline a less-fresh item. This informational asymmetry matters because consumers are demonstrably freshness-aware: willingness to pay for perishable goods falls as freshness deteriorates (Sousa et al. 2025, Tsiros and Heilman 2005). As a result, the same less-fresh item that an in-store shopper would leave on the shelf can instead leave the store as a completed BOPS sale, raising a question that neither the grocer's nor the government's lever was designed to answer: do these two practices still work as intended once BOPS is in place? Concretely, our goal is to study the impact of BOPS on these two practices. By answering this question, we aim to show grocers and policymakers whether and how they should reevaluate their waste-reduction practices in an omnichannel world.

We develop a stylized model of a grocer selling a single perishable product to a market of freshness-aware customers. The product's freshness evolves stochastically on the shelf as a continuous-time Markov chain (CTMC) over three states—fresh, less fresh, and spoiled—and the grocer restocks immediately whenever a unit is sold or spoils. Customers choose between two channels: an in-store channel, in which they observe the product's realized freshness before deciding whether to buy, and a BOPS channel, in which they commit online and receive whatever freshness state is available at pickup. After purchase, an item is carried home, where it may be consumed or may spoil first—generating consumer-side waste alongside the store-side waste that arises when unsold less-fresh units deteriorate on the shelf. The grocer sets the product price and maximizes

long-run average profit. Within this base model, we characterize the equilibrium customer behavior and the optimal price under two configurations: an in-store-only configuration and an omnichannel configuration with BOPS. We then extend the base model in two steps. First, we consider the grocer's adoption of a freshness technology, modeled as a reduction in the store-side spoilage rate, and trace how the resulting equilibrium outcome affects the store-side waste and profit. Second, we consider a government tax levied per unit of store-side waste, which raises the grocer's effective cost of letting products spoil unsold, and examine how the grocer's pricing response to the tax affects the food waste. Together, these analyses isolate how the channel, the freshness technology, and the waste tax each reshape food waste.

Our analysis delivers two main sets of findings. First, the profit and waste implications of freshness technology depend jointly on channel structure and the convenience of in-store shopping. When in-store shopping is convenient (e.g., because of high store density), the store-only retailer realizes a larger profit gain and reduces store waste, whereas the omnichannel retailer realizes a smaller profit gain and may experience an increase in store waste. When in-store shopping is inconvenient (e.g., because of low store density), both comparisons can reverse. Second, a per-unit tax on retail waste weakly reduces store waste but increases household waste, potentially raising total food waste when household spoilage is sufficiently high. Whether the BOPS option mitigates or amplifies this unintended effect depends on online convenience. When BOPS is relatively convenient, the tax-induced increase in total waste is no greater for the omnichannel retailer than for the store-only retailer; when BOPS is less convenient, this comparison can reverse because the tax may induce the omnichannel retailer to use BOPS to shift less-fresh products to consumers. Taken together, these findings show that the effectiveness of retailer and policy interventions depends critically on channel availability and endogenous channel use, and that evaluating interventions solely by retail waste can misstate their true environmental impact.

2. Literature Review

2.1. Food Waste in Grocery Retail Operations

Our work belongs to the broader literature on sustainability in operations management, particularly at the intersection of business-model and operational innovations with environmental and social outcomes (see Girotra and Netessine (2013), Lee and Tang (2018), and Atasu et al. (2020) for reviews). In particular, we study food waste, which the United Nations recognizes as a major sustainability challenge with environmental, economic, and social consequences. Sustainable Development Goal 12.3 calls for halving per-capita food waste at the retail and consumer levels by 2030, while the United Nations Environment Programme documents the substantial scale of food waste and its implications for the global economy, climate, and food security (United Nations General Assembly 2015, UNEP 2024).

One stream of food-waste research examines operational decisions from either a store-side or a customer-side perspective. Store-side studies consider food waste generated by upstream supply-chain decisions and retail operations. At the farm level, Guo et al. (2025) examine how farmers' collaboration with livestream influencers affects selling strategies, profitability, and food waste. Akkaş and Honhon (2022) study how the quantity and age composition of warehouse-to-store shipments determine waste at warehouses and retail stores. At the retail-store level, Hu et al. (2016) study inventory and markdown decisions that balance food disposal against the cannibalization of future regular-price sales, while Iancu et al. (2026) examine how shelf-life choices, inventory-issuance policies, timestamping, and pricing affect sales and in-store waste from premade foods. Customer-side studies focus on food waste in households. Belavina et al. (2017) show that online grocery subscriptions encourage smaller, more frequent purchases and reduce household waste. Zhou et al. (2024) study how residual-shelf-life disclosure and pricing in online grocery affect retailer profit and household food waste. For a broader overview of operations management research on food waste, we refer readers to Akkaş and Gaur (2022).

Another stream of the literature examines how retail decisions affect both store-side and household-side waste. Belavina (2021) shows that greater grocery-store density can reduce household waste while increasing store waste. Wu and Honhon (2023) show that buy-one-get-one-free promotions can reduce retail waste while increasing household waste. Yu and Debo (2026) examine how consumer freshness search interacts with pricing, replenishment, and stock rotation to determine retail and household waste. Yang and Yu (2026) similarly show that surprise clearance (Too Good To Go) can eliminate store waste while increasing consumer waste and may therefore increase or decrease total waste. In a related surplus-retailing setting, Lin et al. (2025) show that pay-what-you-want pricing can increase food waste or reduce food access by changing regular retailers' procurement and donation decisions. Zhou et al. (2026) study how a retailer should determine the number and composition of surprise bags when the realized surplus is uncertain. These studies focus on store density, freshness search, promotions, or surplus retailing rather than regular online grocery channels. Park and Zheng (2026) study inventory and replenishment decisions when in-store customers select fresher products and older inventory can fulfill home-delivery orders. We instead make the common retail price the central decision, endogenize customers' choice between in-store shopping and BOPS, and examine how BOPS changes the profit and waste implications of freshness technology and a store-waste tax.

To the best of our knowledge, our paper is the first to capture both retail-side and consumer-side waste within a single equilibrium framework in which an omnichannel BOPS channel endogenously determines how waste is split between the two. This integrated view is essential to our central message: because BOPS can relocate waste from the shelf to the household rather than eliminate

it, evaluating either side in isolation would misstate the channel's true environmental effect. This concern parallels Long and Gui (2024), who show that apparel waste-reduction strategies can reduce downstream deadstock while increasing upstream fabric acquisition and environmental burdens. Our setting identifies a distinct cross-stage mechanism: BOPS can shift waste downstream from the grocer to the household by changing the freshness of products consumers receive. Methodologically, we capture this two-sided waste through a pair of coupled CTMCs. The first CTMC tracks a unit's freshness on the retail shelf (fresh, less-fresh, or sold/spoiled), while the second tracks the unit's consumption path once it enters a household, where it is ultimately either consumed or wasted. The two chains are linked: the freshness composition of units leaving the store, determined by the grocer's pricing and the customers' channel choices, forms the input to the household chain, so that decisions made at the shelf propagate directly into consumer-side outcomes. This coupled-CTMC formulation lets us characterize store-side and household-side waste jointly and in closed form within a single equilibrium model.

2.2. Omnichannel Retail Management

A substantial body of operations management research has studied omnichannel retail, with particular attention to how adding an online channel affects firm performance and customer behavior. Gallino and Moreno (2014) show that sharing inventory availability information across channels affects online and in-store sales. Bell et al. (2018) examine offline showrooms, showing that allowing customers to inspect products before purchasing generates demand and operational benefits. Gao and Su (2017b) study the role of both product and inventory information in omnichannel retailing, showing how online and offline information interact to shape customer demand. Fransoo et al. (2019) discuss how digital platforms connect neighborhood stores with online ordering and fulfillment. These papers collectively motivate the informational asymmetry at the core of our model: in-store customers can directly inspect the product's freshness state, while BOPS customers commit to purchase without this information.

The BOPS channel has received particular analytical attention. Gao and Su (2017a) develop a stylized model of BOPS operations, showing that BOPS affects customer channel choice through two mechanisms: reducing hassle costs and providing real-time inventory availability information. They establish that not all products are well-suited for BOPS—it is most beneficial for products that underperform in-store—and characterize when BOPS strictly improves retailer profit. Our model builds directly on their framework by introducing perishability and freshness-state dynamics as an additional dimension of the BOPS channel choice problem. Gao and Su (2018) extend the omnichannel service operations framework to settings with self-order technologies. Gao et al. (2022) examine how the introduction of multichannel and omnichannel retailing affects physical

store performance, finding that the relationship between channels is more complex than simple substitution or complementarity. Guo et al. (2026) study BOPS operations on on-demand delivery platforms, extending the BOPS framework to settings where customers can choose between in-store pickup and same-day delivery, a channel configuration increasingly common in practice.

Several papers study specific operational dimensions of omnichannel retail. Nageswaran et al. (2020) examine how cross-channel return policies affect channel choice and retailer profitability. Hu et al. (2022) show that BOPS-induced channel migration can generate either demand pooling or depooling, affecting store fill rates and retailer profitability. Chen et al. (2022) study offline-channel planning in smart omnichannel retailing, examining how retailers should allocate capacity across channels. Baron et al. (2023) identify limitations and operational remedies for omnichannel service integration. Huang et al. (2025) examine how BOPS affects supply chain coordination when supplier encroachment is a concern. Sun et al. (2026) study omnichannel service operations with online order-ahead pickup and on-site ordering, showing that irrevocable advance orders can exacerbate congestion and reduce service throughput.

Our paper contributes to this stream in two ways. First, we introduce product perishability and freshness-state dynamics into the BOPS channel choice framework, a dimension absent from prior omnichannel models. The freshness asymmetry between in-store and BOPS customers—in-store customers inspect and select based on freshness, while BOPS customers receive whatever freshness state is available—creates operational consequences for food waste that existing omnichannel models do not capture. Second, while the omnichannel literature has studied how channel design affects profit and service levels, the environmental implications of channel design decisions—and specifically the displacement of waste from the retail shelf to the consumer household—have not been examined.

3. Model Description

We consider a grocer selling a single perishable product to a market of freshness-aware customers through two channels: a traditional in-store shopping channel and a buy-online-pick-up-in-store (BOPS) channel. These channels represent the predominant grocer-operated fulfillment modes in the grocery industry: physical stores remain the dominant grocery channel, and BOPS has become a widely adopted omnichannel fulfillment method (Brick Meets Click 2025, Food Industry Association and NielsenIQ 2025).[1] In the in-store channel, customers visit the store and directly observe the product's freshness before deciding whether to purchase; in the BOPS channel, customers place

[1] In this paper, we focus on store-based fulfillment in grocery retail and abstract from home delivery, which often relies on third-party providers (Silverstein 2020). Modeling third-party delivery operations is beyond the scope of our analysis and is left for future research.

orders online without observing the product's realized freshness and collect their purchases at the store. We first analyze the in-store-only system as a baseline in Section 3.1, and then study the omnichannel system with both in-store and BOPS channels in Section 3.2.

3.1. Conventional Model: In-Store Channel Only

As a baseline, we first study the case in which only the in-store channel is available. This isolates the role of the BOPS channel in the next section, where we compare outcomes across the two channel configurations.

Retail operations. For simplicity, we assume that the grocer holds at most one unit of product on the shelf and restocks immediately when the item is sold or spoiled, incurring a restocking cost c per unit. We discuss extensions to settings in which the grocer holds multiple units of inventory or restocking is not immediate in Section 6. Our main results remain robust under these scenarios.

The product has three freshness levels: *fresh* (F), *less fresh* (L), and *spoiled*. The freshness evolvement of the product at store is captured by a CTMC with the state space $\{S_F, S_L\}$. Here, S_F means that the product is fresh and S_L implies that the product is less-fresh. The product deteriorates from S_F to S_L and from S_L to *spoiled* exponentially at rate μ (Roccato et al. 2017). When the product is at state S_L, the product leaves the system either through a customer purchase or spoilage. Let λ_F and λ_L denote the purchase rate of the product when it is fresh and less fresh, respectively, which will be characterized later in this section. Then, the effective transition rate from state S_L back to state S_F is $\lambda_L + \mu$. Figure 1 presents the rate diagram that captures the transition of the product's state at store. The transition rate matrix for this CTMC is

$$Q = \begin{bmatrix} -\mu & \mu \\ \lambda_L + \mu & -(\lambda_L + \mu) \end{bmatrix}.$$

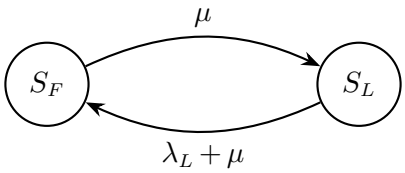


Figure 1 Two-state CTMC for the freshness of the product at store.

Let θ_F and θ_L denote the steady-state distribution that the product at store is in states S_F and S_L. Following from the steady-state conditions $-\mu\theta_F + (\lambda_L + \mu)\theta_L = 0$ and $\theta_F + \theta_L = 1$, we have

$$\theta_F = \frac{\lambda_L + \mu}{\lambda_L + 2\mu}, \quad \theta_L = \frac{\mu}{\lambda_L + 2\mu}. \tag{1}$$

Then, given the product price p, the grocer's long-run average profit function can be expressed as follows:

$$\pi(p) = p(\theta_F\lambda_F + \theta_L\lambda_L) - c(\theta_F\lambda_F + \theta_L\lambda_L + \theta_L\mu) \tag{2}$$

where the first part represents the sales revenue, and the second part is the restocking cost. Note that the grocer restocks the product when either a sale or a spoilage event occurs, and thus the corresponding rate is $\theta_F\lambda_F + \theta_L\lambda_L + \theta_L\mu$.

Customer behavior and utilities. After the customer purchases the product, she may not necessarily consume it immediately. Suppose the customer's consumption time is exponentially distributed with rate ρ. Independently, the product at the customer's home can spoil at an exponentially distributed time with rate μ_c (i.e., the transition rate from F to L and from L to spoiled). Figure 2 displays the rate diagram showing the transition of the product's state at a household, where we use (H_F, H_L, H_C, H_S) to correspond to the state of the product that is fresh, less fresh, consumed, and spoiled at a household.

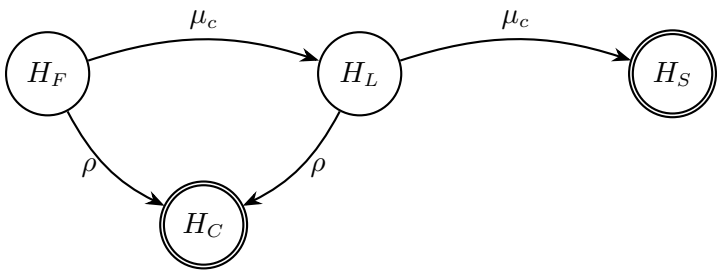


Figure 2 Four-state CTMC for a purchased unit at a household.

The expected valuation of purchasing a less-fresh item, V_L, reflects the probability that the customer consumes it before it spoils as

$$V_L = \left(1 - \frac{\mu_c}{\rho + \mu_c}\right) v_L = \frac{\rho}{\rho + \mu_c} v_L, \tag{3}$$

where v_L is the value of consuming a less-fresh item. If the customer purchases a fresh item, then she either consumes while the product is fresh and gets $v_F > v_L$ or the product first becomes less fresh, after which her expected valuation is V_L. Thus, given the transition rates in Figure 2, the expected valuation of purchasing a fresh item, V_F, is as follows:

$$V_F = \frac{\rho}{\rho + \mu_c} v_F + \frac{\mu_c}{\rho + \mu_c} V_L = \frac{\rho}{\rho + \mu_c} v_F + \frac{\mu_c \rho}{(\rho + \mu_c)^2} v_L. \tag{4}$$

Since $v_F > v_L$, we have $V_F > V_L$. When consumers are in the store, they are able to observe the freshness of the product, and can make a purchase decision accordingly: If the product is fresh (i.e., at state S_F or H_F), then she buys the product if and only if the expected valuation is greater or equal to the product price, i.e., $V_F \geq p$; similarly, if the product is less fresh (i.e., S_L or H_L), then she buys the product if and only if $V_L \geq p$.

When deciding whether to visit the store, customers base their decisions on the expected valuation derived from the product's freshness at the time of purchase, where the expectation is taken

with respect to the stationary distribution of the product's freshness levels. The utility of visiting the store is then given by

$$u_s = -h_s + \theta_F(V_F - p)^+ + \theta_L(V_L - p)^+, \tag{5}$$

where $(\cdot)^+ = \max(\cdot, 0)$ and $h_s > 0$ is the hassle cost associated with an in-store purchase, such as the travel cost and the cost of searching for the item in the store. A customer visits the store if and only if $u_s \geq 0$.

Denote the customer arrival rate as Λ. Then, based on consumers' store visiting decision and their purchase decision within the store described above, we can obtain the demand rate of the product when it is fresh (λ_F) or less fresh (λ_L) as follows:

$$\lambda_F = \Lambda \mathbf{1}\{u_s \geq 0 \text{ and } V_F - p \geq 0\}, \quad \lambda_L = \Lambda \mathbf{1}\{u_s \geq 0 \text{ and } V_L - p \geq 0\}. \tag{6}$$

where $\mathbf{1}\{\cdot\}$ denotes the indicator function. In the main model, consumers are homogeneous, and thus the demand rate is governed simply by an indicator function. This homogeneous demand setting is consistent with Gao and Su (2017a). In Section 6.3, we extend the model to customers with heterogeneous hassle costs. Our main insights remain valid under this model extension.

A modeling assumption implicit in Equation (6) is that the customer arrival rate Λ does not depend on the customer's consumption rate μ_c: how frequently a customer arrives to consider buying the product is not governed by how quickly they consume the units they have already purchased at home. We emphasize that Λ is the rate of potential purchase consideration, not the realized demand rate; the realized demand rates λ_F and λ_L in Equation (6) are determined endogenously by which arriving customers choose to buy, given the price and the freshness state they encounter. The independence of Λ and μ_c is justified by the separation of timescales between purcharse consideration and at-home consumption. Survey evidence indicates that U.S. consumers generally shop for groceries roughly weekly, with most shopping at least weekly and nearly all at least monthly (Brenan 2022, Hayes 2024). A customer may consider purchasing the focal product (e.g., lettuce) on only some grocery trips, so the time between opportunities to consider it can be substantially longer than the time between grocery trips. By comparison, once purchased, an individual unit is typically consumed or spoils within a few days: leafy greens such as spinach, romaine, and lettuce retain quality for roughly three to seven days under refrigeration (Feast Local 2023). Because the time between a customer's successive purchase occasions substantially exceeds the time it takes them to consume a unit at home, the consumption rate μ_c is effectively decoupled from the rate at which the customer returns to the store to buy the product. In other words, the purchase cycle is governed by broader shopping habits rather than by household consumption dynamics. We therefore treat Λ as a primitive independent of μ_c.

Considering the behaviors of customers, the grocer's profit maximization problem can be expressed as

$$\pi^C = \max_{p \geq 0} \pi(p) \quad \text{s.t. Equations } (1)-(6).$$

We use the superscript $\cdot^C$ to denote the equilibrium outcome in the conventional model.

LEMMA 1. *In the conventional model, the grocer's profit in equilibrium is positive if and only if* $h_s < \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$ *and* $c < \max\left\{\frac{\Lambda(V_F-2h_s)}{\Lambda+\mu}, \frac{\Lambda(\Lambda+2\mu)}{(\Lambda+\mu)^2}\min\left\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}-h_s\right\}\right\}$. *Under this condition, the equilibrium product price* p^C *and the associated profit* π^C *are characterized as follows.*

(i) If $h_s \leq \min\left\{\frac{V_F-V_L}{2}, \frac{V_F-2V_L}{2}+\frac{c(\Lambda+\mu)}{2(\Lambda+2\mu)}\right\}$, *then* $p^C = V_F - 2h_s$ *and* $\pi^C = \frac{\Lambda}{2}(V_F-2h_s-c) - c\frac{\mu}{2}$.

(ii) If $h_s > \min\left\{\frac{V_F-V_L}{2}, \frac{V_F-2V_L}{2}+\frac{c(\Lambda+\mu)}{2(\Lambda+2\mu)}\right\}$, *then* $p^C = \min\left\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}-h_s\right\}$ *and* $\pi^C = \Lambda\left(p^C-c\right) - \frac{c\mu^2}{\Lambda+2\mu}$.

Lemma 1 shows that the optimal price traces out two pricing regimes as h_s varies. When hassle is low (case (i)), the grocer charges a premium price $V_F - 2h_s$ that only fresh units clear, deliberately leaving less-fresh units to spoil unsold: low hassle keeps customers willing to pay up for fresh product, so serving fresh-only demand at a high margin dominates. As hassle rises (case (ii)), the grocer can no longer sustain the premium and instead lowers the price to sell both freshness levels, charging $p^C = \min\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}-h_s\}$. For the rest of the analysis, we assume $h_s \leq \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$ and $c < \max\left\{\frac{\Lambda(V_F-2h_s)}{\Lambda+\mu}, \frac{\Lambda(\Lambda+2\mu)}{(\Lambda+\mu)^2}\min\left\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}-h_s\right\}\right\}$ to ensure positive profit for the grocer in this benchmark case.

3.2. Omnichannel Model: BOPS

We now consider an omnichannel system that the grocer operations on both store and BOPS channels. Consistent with common practice in industry (Hogan et al. 2026) and the relevant literature on omnichannel retailing (Gao and Su 2017a), the grocer sets a single price p in both channels.

For the customer who decides to shop in store, her utility function u_s remains same as defined in Equation (5). If the customer who decides to order via BOPS, she commits to a purchase without observing the realized freshness level, and receives the product regardless of its freshness state at the time of fulfillment. Thus, her utility is

$$u_b = -h_b + \theta_F V_F + \theta_L V_L - p, \tag{7}$$

where $h_b > 0$ is the hassle cost of purchasing through the BOPS channel. Following Gao and Su (2017a), we parameterize the relationship between the hassle costs h_s and h_b as

$$h_b = \alpha h_s + h_o, \quad \alpha \in (0,1), \tag{8}$$

where h_o corresponds to the hassle cost of placing an order online, and αh_s reflects the fact that the customer must still travel to the store to pick up the product but need not search for it in the store. Assume $h_o < (1-\alpha)h_s$; otherwise, no customer will ever consider the BOPS option as $u_s > u_b$ holds for any θ_F, θ_L, p.

Customers choose the channel that yields a higher non-negative utility. If a customer decides to proceed with the store channel, her behavior is the same as described in Section 3.1. Otherwise, she commits a purchase via the BOPS channel regardless of the product's freshness level. Recall that the customer arrival rate is Λ. With the price p, the demand rate of the product whose freshness level is fresh (λ_F) and less fresh (λ_L) as follows:

$$\lambda_F = \Lambda \mathbf{1}\{u_s(p) > u_b(p) \text{ and } u_s \geq 0 \text{ and } V_F - p \geq 0\} + \Lambda \mathbf{1}\{u_b(p) \geq \max\{u_s(p), 0\}\}, \tag{9}$$

$$\lambda_L = \Lambda \mathbf{1}\{u_s(p) > u_b(p) \text{ and } u_s \geq 0 \text{ and } V_L - p \geq 0\} + \Lambda \mathbf{1}\{u_b(p) \geq \max\{u_s(p), 0\}\}. \tag{10}$$

Note that all inventory is still fulfilled through the store channel. Therefore, given a common price across both channels, the grocer's inventory restocking cost and revenue function remain unchanged, and the profit function is therefore the same as that in Equation (2). Accordingly, the grocer's profit-maximization problem can be written as

$$\pi^B = \max_{p \geq 0} \pi(p) \quad \text{s.t. Equations } (1)-(4) \text{ and } (7)-(10).$$

We use the superscript $\cdot^B$ to denote the equilibrium outcome in the omnichannel model with BOPS.

LEMMA 2. *There exists threshold $\mathcal{H}_o \leq (1-\alpha)h_s$ such that*

(i) if $h_o < \mathcal{H}_o$, then it is strictly profitable for the grocer to offer BOPS to customers (i.e., $\pi^B > \pi^C$). Under this condition, the equilibrium price is $p^B = \min\left\{\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_b, V_L + \frac{\Lambda+2\mu}{\mu}(h_s - h_b)\right\}$ and the associated profit $\pi^B = \Lambda(p^B - c) - \frac{c\mu^2}{\Lambda+2\mu}$.

(ii) if $h_o \geq \mathcal{H}_o$, the grocer does not offer BOPS, and consequently the equilibrium is identical to that in the conventional model.

Lemma 2 characterizes when offering BOPS is profitable and how the grocer prices when it does. The decision hinges on the online ordering cost h_o. When $h_o < \mathcal{H}_o$, online ordering is sufficiently convenient to offset the freshness risk of committing sight-unseen, making BOPS strictly profitable for the grocer. As h_o increases, however, the benefit of BOPS diminishes. Once $h_o \geq \mathcal{H}_o$, offering BOPS is no longer profitable, and the grocer reverts to the conventional in-store-only outcome.

4. Impact of Reducing the Store Spoilage Rate

A grocer has several in-store levers for slowing how quickly fresh products deteriorate on its shelves, such as refrigerated and humidity-controlled display cases, produce misting, and temperature monitoring (Kader 2002, Rizzo 2025, Zuo et al. 2025). Across these levers the choice is largely discrete—the grocer either invests in the freshness measure or does not—so we model it as a switch from a baseline store spoilage rate μ without the technology to a lower rate $\mu' < \mu$ with it, and examine how this reduction affects the grocer's profit and the resulting store-side waste.

Recall that store-side waste is generated whenever a less-fresh unit spoils in store, which occurs at rate μ. For a given μ, the store-side waste rate is therefore

$$W_s(\mu) = \theta_L \mu. \tag{11}$$

Let $\pi_\mu(p)$ denote the gross profit function when the grocer's store spoilage rate is μ, excluding the costs associated with achieving and maintaining that spoilage rate. Modifying the profit function in Equation (2) to make the store-waste term explicit, we write

$$\pi_\mu(p) = (p-c)(\theta_F \lambda_F + \theta_L \lambda_L) - cW_s(\mu),$$

where the first term is the profit from sales and the second is the restocking cost paid for spoiled units. Let π_μ^C and π_μ^B denote the grocer's optimal gross profit under the conventional and omnichannel models, respectively:

$$\begin{aligned} \pi_\mu^C &= \max_{p \geq 0} \pi_\mu(p) \quad \text{s.t. Equations } (1)-(6); \\ \pi_\mu^B &= \max_{p \geq 0} \pi_\mu(p) \quad \text{s.t. Equations } (1)-(4) \text{ and } (7)-(10). \end{aligned}$$

Suppose the grocer lowers its store spoilage rate from μ to μ'. Proposition 1 characterizes the resulting change in the grocer's profit.

Proposition 1. *In both the conventional and omnichannel models, the optimal gross profit weakly increases when the in-store spoilage rate decreases from μ to μ', i.e., $\pi_\mu^C \leq \pi_{\mu'}^C$ and $\pi_\mu^B \leq \pi_{\mu'}^B$.*

Proposition 1 is intuitive. Lowering the spoilage rate from μ to μ' helps the grocer through two different mechanism. First, it directly reduces the less-fresh units that spoil unsold, lowering spoilage-driven restocking costs. Second, it raises the chance an arriving customer finds a fresh unit and supports a higher price. Since both effects increase profitability, the optimal gross profit weakly increases as the spoilage rate decreases. The practical reading is that freshness technology is not merely an environmental amenity adopted out of sustainability concern—it is a profit-improving investment in its own right.

Although the optimal profit weakly increases as the store's spoilage rate falls, the magnitude of this gain may differ across the conventional and omnichannel models. To compare them, define Δ_{π^C} and Δ_{π^B} as the profit increase under the conventional and omnichannel models, respectively, when μ decreases to μ':

$$\Delta_{\pi^C} = \pi^C_{\mu'} - \pi^C_{\mu}, \quad \Delta_{\pi^B} = \pi^B_{\mu'} - \pi^B_{\mu}.$$

Proposition 2 characterizes how store-side waste and the magnitude of the profit gain change when μ decreases to μ'.

PROPOSITION 2. *There exists a threshold $\mathcal{H}_s$, such that*

(i) If $h_s < \mathcal{H}_s$, then

- *In the conventional model, store waste always decreases when μ decreases to μ', i.e., $W_s^C(\mu) \geq W_s^C(\mu')$.*
- *In the omnichannel model, store waste increases when μ decreases to μ' (i.e., $W_s^B(\mu) \leq W_s^B(\mu')$) if $h_o \in (\mathcal{H}_{o1}, \mathcal{H}_{o2})$ and $\mu' \in (\mathcal{M}_b, \mu)$ for some thresholds $\mathcal{H}_{o1}, \mathcal{H}_{o2}$, and $\mathcal{M}_b$; under this condition, $\Delta_{\pi^B} < \Delta_{\pi^C}$.*

(ii) If $h_s \geq \mathcal{H}_s$, then

- *In the conventional model, store waste increases when μ decreases to μ' (i.e., $W_s^C(\mu) \leq W_s^C(\mu')$) if $\mu' \in (\mathcal{M}_{c1}, \mathcal{M}_{c2})$ for some thresholds $\mathcal{M}_{c1}$ and $\mathcal{M}_{c2}$.*
- *In the omnichannel model, store waste decreases when μ decreases to μ' (i.e., $W_s^B(\mu) \geq W_s^B(\mu')$) if $h_o < \mathcal{H}_{o3}$ for some threshold $\mathcal{H}_{o3}$; under this condition, $\Delta_{\pi^B} > \Delta_{\pi^C}$ if $\mu' \in (\mathcal{M}_{c1}, \mathcal{M}_{c2})$ and $h_o \geq \mathcal{H}_{o4}$ for some threshold $\mathcal{H}_{o4}$.*

Proposition 2 shows that both the waste consequence of slowing spoilage and the value of doing so depend on how convenient the store is (captured by h_s relative to $\mathcal{H}_s$) and on whether the grocer offers BOPS.

When the store is convenient (case (i), $h_s < \mathcal{H}_s$), a conventional grocer's store waste falls unambiguously as μ decreases, a direct and intuitive consequence of slower spoilage. In the omnichannel model, however, store waste can increase because the freshness improvement may induce an endogenous change in the grocer's optimal channel strategy. A lower spoilage rate makes fresh-only in-store selling more profitable and can reduce the grocer's incentive to offer BOPS. This switch occurs when the online hassle is moderate, $h_o \in (\mathcal{H}_{o1}, \mathcal{H}_{o2})$: if h_o is low, the grocer continues to offer BOPS after the improvement, whereas if it is very high, the grocer does not offer BOPS even before the improvement. In this intermediate region, the grocer offers BOPS at μ but stops offering it at μ', shifting freshness-aware customers from the online channel back to the store, where they inspect and reject less-fresh units. Consequently, when the reduction in spoilage is modest, the additional

less-fresh units left unsold can outweigh the direct benefit of slower spoilage, causing store waste to rise and yielding $\Delta_{\pi B} < \Delta_{\pi C}$.

When the store is inconvenient (case (ii), $h_s \geq \mathcal{H}_s$), the mechanism is different and the comparison may reverse. For a conventional grocer, the waste change is driven by a shift in pricing strategy. When the store is inconvenient, hassle already depresses customers' effective valuations, so the margin the grocer earns by clearing less-fresh units at a low price is thin. As μ decreases and the shelf becomes reliably fresh, the grocer feels less pressure to clear less-fresh units at a low price and instead moves toward charging a high price and selling only fresh units, leaving more less-fresh stock to spoil, so store waste can rise. Note that this switch only occurs when the store is inconvenient: when h_s is small (case (i)), the grocer's less-fresh margin is large enough that it continues to sell both fresh and less-fresh units as μ falls, and store waste simply declines. In addition, the store waste increase in the conventional model also requires the improvement by the freshness technology to be moderate ($\mu' \in (\mathcal{M}_{c1}, \mathcal{M}_{c2})$: if μ' is too close to μ (a small improvement), the shelf is not fresh enough to justify giving up the revenue from selling less-fresh units and no switch occurs; if μ' is too far below μ (a drastic improvement), the shelf becomes so reliably fresh that less-fresh units rarely arise at all, and waste falls again. Only in the intermediate band does the grocer switch to selling fresh units only while still leaving enough less-fresh stock to spoil.

An omnichannel grocer is not forced into this corner, provided the online channel is genuinely usable ($h_o < \mathcal{H}_{o3}$): it has an alternative outlet for less-fresh units, namely pushing them to customers who commit online, and so need not lean on the high-price, low-volume strategy, particularly when the store is inconvenient and customers are already inclined toward BOPS. Store waste therefore decreases from μ to μ'. Moreover, when $\mu' \in (\mathcal{M}_{c1}, \mathcal{M}_{c2})$, waste decreases under BOPS but increases in the conventional model. Because of this different effect on waste, the profit benefit of improving freshness can be higher under BOPS (i.e., $\Delta_{\pi B} > \Delta_{\pi C}$), especially when the online hassle is not too small ($h_o \geq \mathcal{H}_{o4}$), so that improving product freshness has a greater impact on the omnichannel grocer's pricing power.

The two cases above are distinguished by the in-store hassle cost h_s, which can be interpreted as a proxy for store density. The hassle cost captures the time, travel, and effort a customer expends to shop in person; this burden is smaller where stores are dense and easy to reach, and larger where they are sparse. A grocer with high store density—a dense urban chain whose outlets customers pass frequently and reach with little effort—therefore corresponds to a low h_s (a convenient store, case (i)), while a grocer with low store density—a suburban or rural operator whose stores customers reach only with a dedicated trip—corresponds to a high h_s (an inconvenient store, case (ii)). Reading h_s this way, Proposition 2's two cases map onto two recognizable retail environments, which yields two broader implications.

First, on waste. The effect of a freshness investment on in-store waste depends jointly on store density and channel structure, and the two formats may respond in opposite ways across the density spectrum. For a high-density grocer, lowering (μ) reduces store waste under the conventional model, as expected; under the omnichannel model, however, the resulting fresher shelf may induce freshness-sensitive customers to switch from BOPS to in-store shopping, where they inspect and reject less-fresh units, leaving those units to spoil and potentially increasing store waste. For a low-density grocer, the pattern reverses: lowering (μ) can increase store waste under the conventional model because the grocer may maintain a high price and leave less-fresh units unsold. Under the omnichannel model, however, offering BOPS can reduce store waste by routing less-fresh units to online customers rather than leaving them to spoil on the shelf. The managerial point is that a freshness investment is not uniformly advantageous: its waste consequence may flip with both the grocer's store density and its channel structure, so the same technology that cuts shelf waste for one grocer can raise it for another.

Second, on profit and technology adoption. Our model characterizes the gross-profit benefit of improving freshness but does not explicitly incorporate the cost of adopting freshness technology. The technology is therefore attractive when the resulting profit gain exceeds its implementation cost. Under this interpretation, the business case for freshness technology depends jointly on the grocer's channel structure and store density. For a high-density grocer, improving freshness generates a larger profit gain under the conventional model than under the otherwise identical omnichannel model, making the investment relatively more attractive without BOPS. For a low-density grocer, however, this comparison may reverse: the omnichannel grocer may realize a larger profit gain because it can better leverage fresher inventory through its online channel, making adoption relatively more attractive with BOPS. Consequently, two otherwise similar grocers evaluating the same freshness technology may rationally make different adoption decisions because of differences in their channel structures and store densities.

5. Impact of Retail Waste Tax

As discussed in Section 1, governments increasingly act on the disposal side to curb food waste, effectively raising the marginal cost a grocer bears for every unit that spoils unsold. We capture this common feature in reduced form as a per-unit waste tax, where T denotes the tax charged per spoiled unit. To incorporate this cost, let $\pi_T(p)$ denote the profit function when the government charges T per unit of spoiled product. Modifying the profit function in Equation (2), we have

$$\pi_T(p) = (p-c)(\theta_F\lambda_F + \theta_L\lambda_L) - (c+T)W_{s,T}, \tag{12}$$

where $W_{s,T} = \mu\theta_L$ is the store-side waste rate. The first term in Equation (12) captures the profit from sales, while the second captures the total cost paid for spoiled units, comprising both the

restocking cost and the waste tax. Note that although the store-side waste rate $W_{s,T}$ does not contain T in its expression directly, the tax shapes store-side waste indirectly through the grocer's pricing response. A higher T raises the effective penalty on each spoiled unit, which makes clearing less-fresh units through sales relatively more attractive than letting them spoil. By adjusting its price—and, under BOPS, how it shifts customers between the in-store and online channels—the grocer changes the purchase rate λ_L of less-fresh units. Since the steady-state composition $\theta_L = \frac{\mu}{\lambda_L+2\mu}$ depends on λ_L, a tax-induced increase in λ_L lowers θ_L and the store-side waste rate $W_{s,T}$.

We then let π_T^C and π_T^B denote the grocer's optimal profit under the conventional and omnichannel models, respectively, when the unit tax is T:

$$\pi_T^C = \max_{p\geq 0} \pi_T(p) \quad \text{s.t. Equations } (1)-(6);$$
$$\pi_T^B = \max_{p\geq 0} \pi_T(p) \quad \text{s.t. Equations } (1)-(4) \text{ and } (7)-(10).$$

We restrict attention to tax levels for which the grocer's optimal operating profit remains positive under both channel. Evaluating the effectiveness of a waste tax requires a broader measure of waste than the store-side analysis of the previous section. There, our focus was the grocer's own profit and the waste visible on its shelf, both of which are fully captured by store-side quantities. A waste tax, however, is a policy instrument whose stated purpose is to reduce *total* food waste, not merely to clean up the retail shelf. As we will show, the tax can induce the grocer to clear less-fresh units by routing them to customers, which lowers store-side waste while shifting it downstream. Judging the tax by store-side waste alone would therefore miss precisely the effect of interest—whether the policy eliminates waste or merely relocates it. To assess this, we must track waste on the customer side as well.

After purchase, customer-side waste occurs in two ways: a fresh unit deteriorates to less-fresh and then spoils at home before consumption, while a less-fresh unit becomes waste directly upon spoiling. The customer-side waste therefore depends on the freshness level of the product the customer receives at purchase. Let $W_{c,T}$ denote the customer-side waste when the unit tax is T:

$$W_{c,T} = \lambda_F \left(\frac{\mu_c}{\rho+\mu_c}\right)^2 \theta_F + \lambda_L \frac{\mu_c}{\rho+\mu_c}\theta_L. \tag{13}$$

To see how this expression arises, consider a unit once it has been taken home, where it is consumed at rate ρ and deteriorates at the consumer-side spoilage rate μ_c. At any single freshness stage, these two processes compete, and the deterioration process fires first—the unit loses the race to consumption and moves one step toward spoiling—with probability $\frac{\mu_c}{\rho+\mu_c}$. The two terms correspond to the two freshness states in which a unit may be purchased. A unit purchased less-fresh (the second term, arriving at rate λ_L and received less-fresh with probability θ_L) is a single stage away

from spoiling, so it becomes waste with probability $\frac{\mu_c}{\rho+\mu_c}$. A unit purchased fresh (the first term, arriving at rate λ_F and received fresh with probability θ_F) must instead lose this race twice—first deteriorating from fresh to less-fresh, then spoiling from less-fresh—so it becomes waste with probability $(\frac{\mu_c}{\rho+\mu_c})^2$. Summing the two contributions, weighted by the rate and freshness composition at which each type of unit is purchased, yields $W_{c,T}$.

As before, we use $\cdot^C$ and $\cdot^B$ to denote the equilibrium outcomes of the conventional and omnichannel models, respectively. Proposition 3 characterizes how store-side and customer-side waste change once the tax is imposed.

PROPOSITION 3. *In both the conventional and omnichannel models, imposing tax weakly decreases the store waste (i.e., $W^C_{s,T} \le W^C_{s,0}$ and $W^B_{s,T} \le W^B_{s,0}$) and weakly increases the customer waste (i.e., $W^C_{c,T} \ge W^C_{c,0}$ and $W^B_{c,T} \ge W^B_{c,0}$).*

Proposition 3 shows that the tax does reduce store-side waste, but only by relocating it downstream to the household. The mechanisms in the two systems are different. In the conventional model, the tax raises the effective cost of letting less-fresh units spoil unsold, so the grocer lowers its price to clear those units through sales rather than discard them, pushing the waste onto customers who may not consume them in time. In the omnichannel model, the grocer has an additional lever: it can steer customers toward the online channel and fulfill their committed orders with less-fresh units, again converting shelf waste into household waste. The practical implication is that a government evaluating a waste tax cannot judge its effectiveness by store-side figures alone—the tax may clean up the shelf while pushing the wastes to the household.

Whether this displacement reduces total waste, however, depends on how the two opposing movements net out, which we examine next. Combining the two components, the total waste is $W_{t,T} = W_{s,T} + W_{c,T}$. We define Δ_{W^C} and Δ_{W^B} as the change in total waste under the conventional and omnichannel models after the tax is applied, respectively: $\Delta_{W^C} = W^C_{t,T} - W^C_{t,0}$ and $\Delta_{W^B} = W^B_{t,T} - W^B_{t,0}$. Proposition 4 characterizes how total waste changes after the tax is applied.

PROPOSITION 4. *In both the conventional and omnichannel models, imposing tax weakly increases the total waste if and only if $\mu_c \ge \bar{\mu}_c$ for some threshold $\bar{\mu}_c$. Under this condition,*

(i) if $h_o < \mathcal{H}_o$, then $\Delta_{W^B} = 0 \le \Delta_{W^C}$;

(ii) if $h_o \ge \mathcal{H}_o$, then $\Delta_{W^B} \ge \Delta_{W^C}$,

where $\mathcal{H}_o$ is defined in Lemma 2.

As a result of the waste displacement from grocers to consumers shown in Proposition 3, Proposition 4 shows that the waste tax, designed to curb food waste, can in fact *increase* total waste, especially when the consumer spoilage rate is high (i.e., $\mu_c \ge \bar{\mu}_c$). More importantly, BOPS and the corresponding online shopping hassle cost h_o shape when and how severely backfire occurs.

Although easier online ordering might be expected to amplify the tax-induced displacement of less-fresh products to consumers, the model yields the opposite comparison. When $h_o < \mathcal{H}_o$, tconsumers already find BOPS sufficiently convenient and shop online before the tax, leaving little scope for the tax to induce a further shift from in-store to online demand; hence, $\Delta_{W^B} = 0 \leq \Delta_{W^C}$. When $h_o \geq \mathcal{H}_o$, BOPS is not offered initially. Once the tax makes shelf waste more costly, however, the grocer can activate BOPS to move less-fresh products to consumers without relying solely on price cuts, whereas a conventional grocer has only the less-profitable option of discounting these products. The omnichannel grocer may therefore redirect less-fresh products in cases where the conventional grocer would leave them on the shelf. When household spoilage is sufficiently high, this greater displacement to consumers makes the tax-induced increase in total waste weakly larger under the omnichannel model, so $\Delta_{W^B} \geq \Delta_{W^C}$. Thus, the tax's unintended effects may be more pronounced when online ordering is less convenient but remains a viable outlet for redirecting less-fresh products.

The policy implication is that a uniform waste tax can be a blunt instrument in an omnichannel market because its effect on total waste depends on how grocers adjust their sales across channels. A government that evaluates the policy solely by reductions in store waste may be misled, as cleaner shelves may reflect the displacement of less-fresh products to consumers rather than a genuine reduction in waste. The same tax may therefore have different total-waste consequences depending on the extent to which it encourages grocers to expand BOPS sales. Tax design and evaluation in omnichannel settings should account for such adjustments in grocers' channel mix and monitor whether reductions in store waste are offset by increased household waste.

6. Model Extensions

6.1. Shelf Size

In our main model, the grocer stocks a single unit of the product on the shelf at a time. In this section we extend the model to a larger shelf that holds up to two units of the same product, restocking each immediately when it is sold or spoiled. Because the two units are identical and enter symmetrically, it suffices to track how many of the two on-shelf units are fresh versus less-fresh. We therefore describe the shelf by a CTMC with state space $\{S_{(F,F)}, S_{(F,L)}, S_{(L,L)}\}$, where the state $S_{(\cdot,\cdot)}$ records the freshness levels of the two units: both fresh ($S_{(F,F)}$), one fresh and one less-fresh ($S_{(F,L)}$), or both less-fresh ($S_{(L,L)}$). This extension broadens the model beyond a single unit of product: with two units sharing the shelf, their freshness states evolve jointly, so the grocer's store waste and channel decisions respond to the full shelf composition rather than the freshness of a single unit. A larger shelf also sharpens the informational asymmetry between the two channels: unlike in the single-unit model, the two channels now induce different shelf dynamics, which we characterize separately below.

6.1.1. Conventional Model Each fresh unit deteriorates to less-fresh independently at rate μ, so the system moves from $S_{(F,F)}$ to $S_{(F,L)}$ at rate 2μ (either fresh unit may deteriorate) and from $S_{(F,L)}$ to $S_{(L,L)}$ at rate μ (the remaining fresh unit deteriorates).

An in-store customer inspects both units and, whenever a fresh unit is available, selects it, accepting a less-fresh unit only when both units are less-fresh. Thus, in the state $S_{(F,L)}$, the single less-fresh will never be purchased and leaves the system only by spoilage, so a transition from $S_{(F,L)}$ to $S_{(F,F)}$ occurs at rate μ. In state $S_{(L,L)}$, a less-fresh unit is replaced when either unit spoils (rate 2μ) or when an arriving customer buys one (rate λ_L), so a transition from $S_{(L,L)}$ to $S_{(F,L)}$ occurs at rate $\lambda_L + 2\mu$. The rate diagram is shown in Figure 3 and the transition rate matrix is

$$Q = \begin{bmatrix} -2\mu & 2\mu & 0 \\ \mu & -2\mu & \mu \\ 0 & \lambda_L + 2\mu & -(\lambda_L + 2\mu) \end{bmatrix}.$$

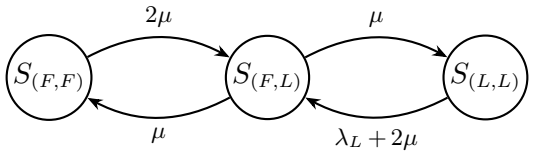


Figure 3 Three-state CTMCs for the joint freshness of two units of the same product sharing the shelf under the conventional model.

Let $\theta_{(F,F)}$, $\theta_{(F,L)}$, and $\theta_{(L,L)}$ denote the steady-state shelf probabilities. This is a birth–death chain, so solving the balance equations together with the normalization $\theta_{(F,F)} + \theta_{(F,L)} + \theta_{(L,L)} = 1$ yields

$$\theta_{(F,F)} = \frac{\lambda_L + 2\mu}{3\lambda_L + 8\mu}, \quad \theta_{(F,L)} = \frac{2(\lambda_L + 2\mu)}{3\lambda_L + 8\mu}, \quad \theta_{(L,L)} = \frac{2\mu}{3\lambda_L + 8\mu}. \tag{14}$$

Because an in-store customer selects a less-fresh unit only when both on-shelf units are less-fresh, her utility from visiting the store is

$$u_s = -h_s + \big(\theta_{(F,F)} + \theta_{(F,L)}\big)(V_F - p)^+ + \theta_{(L,L)}(V_L - p)^+, \tag{15}$$

since a fresh unit is present in states $S_{(F,F)}$ and $S_{(F,L)}$, and only in $S_{(L,L)}$ must she accept a less-fresh unit. The realized demand rates follow the same expression as in Equation (6).

To express the grocer's profit, we compute the per-unit-time sales, store waste, and restocking rates. A fresh unit is sold whenever a customer arrives and at least one fresh unit is on the shelf—that is, in states $S_{(F,F)}$ and $S_{(F,L)}$—so fresh units are sold at rate $\lambda_F(\theta_{(F,F)} + \theta_{(F,L)})$. A less-fresh unit is sold only in state $S_{(L,L)}$, and at most one unit per arrival, so less-fresh units are sold at rate

$\lambda_L\theta_{(L,L)}$. Store waste accrues at rate μ for each less-fresh unit on the shelf, so the store-waste rate is $\mu\theta_{(F,L)} + 2\mu\theta_{(L,L)}$. Therefore, for a given price p, the grocer's long-run average profit function is

$$\begin{aligned}\pi(p) = p\left[\lambda_F(\theta_{(F,F)} + \theta_{(F,L)}) + \lambda_L\theta_{(L,L)}\right] \\ - c\left[\lambda_F(\theta_{(F,F)} + \theta_{(F,L)}) + \lambda_L\theta_{(L,L)} + \mu\theta_{(F,L)} + 2\mu\theta_{(L,L)}\right],\end{aligned} \tag{16}$$

where the first term is sales revenue and the second is the total restocking cost incurred on both sold and spoiled units. The grocer's profit-maximization problem is

$$\pi^C = \max_{p\geq 0} \pi(p) \quad \text{s.t. Equations (14), (16), (3) − (4), (15), and (6).}$$

6.1.2. Omnichannel Model Unlike an in-store customer, who inspects the available units and selects the freshest one, a BOPS customer commits to a purchase before fulfillment and receives a unit selected by the store's picker. Store-based online fulfillment prioritizes picking speed over unit-level freshness, so pickers generally select an accessible unit without carefully evaluating its freshness (McKinsey & Company 2022, The Philadelphia Inquirer 2020). Motivated by these fulfillment practices, we assume that, when one fresh and one less-fresh unit are available, the picker selects the fresh unit with probability $\psi \in [0,1]$ and the less-fresh unit with probability $1-\psi$. The probability ψ captures randomness or unobserved heterogeneity in which unit is ultimately assigned to a BOPS order, conditional on the shelf containing one unit of each freshness level. We treat ψ as exogenous: it summarizes factors outside the grocer's modeled decisions, such as the physical layout of the shelf, the positioning and accessibility of units, and idiosyncratic picker behavior. A BOPS customer receives a fresh unit with probability $\theta_{(F,F)} + \psi\theta_{(F,L)}$ and a less-fresh unit with probability $\theta_{(L,L)} + (1-\psi)\theta_{(F,L)}$. Her utility is therefore

$$u_b = -h_b + \left(\theta_{(F,F)} + \psi\theta_{(F,L)}\right) V_F + \left(\theta_{(L,L)} + (1-\psi)\theta_{(F,L)}\right) V_L - p. \tag{17}$$

Unlike the main model, where a single less-fresh purchase rate λ_L suffices, the two-unit shelf forces us to distinguish the two channels' purchase rates, because a fresh and a less-fresh unit can now sit on the shelf together (state $S_{(F,L)}$) and the two channels act on that shared less-fresh unit in different ways. An in-store customer inspects both units, takes the fresh one, and leaves the less-fresh unit to spoil; a BOPS customer, committed before fulfillment and handed one of the two units at random, clears the less-fresh unit with probability $1-\psi$. The rate at which less-fresh units leave the shelf therefore depends on which channel a customer uses, so a single λ_L can no longer describe the shelf dynamics. To capture this, we let $\lambda_{F,s}$ and $\lambda_{L,s}$ denote the rates at which in-store customers purchase fresh and less-fresh units, respectively, and let λ_o denote the rate at which BOPS customers purchase units of either freshness. Conditional on BOPS being offered,

$$\lambda_{F,s} = \Lambda\mathbf{1}\{u_s(p) > u_b(p) \text{ and } u_s \geq 0 \text{ and } V_F - p \geq 0\}, \tag{18}$$

$$\lambda_{L,s} = \Lambda\mathbf{1}\{u_s(p) > u_b(p) \text{ and } u_s \geq 0 \text{ and } V_L - p \geq 0\}, \tag{19}$$

$$\lambda_o = \Lambda\mathbf{1}\{u_b(p) \geq \max\{u_s(p), 0\}\}. \tag{20}$$

In state $S_{(F,L)}$, an in-store customer always takes the fresh unit and never the less-fresh one, whereas a BOPS customer receives the less-fresh unit with probability $1-\psi$. The less-fresh unit is therefore purchased online at rate $(1-\psi)\lambda_o$ and may additionally spoil at rate μ, so a transition from $S_{(F,L)}$ to $S_{(F,F)}$ occurs at rate $\mu+(1-\psi)\lambda_o$. In state $S_{(L,L)}$, both units are less fresh, so any purchase (in-store or online) removes a less-fresh unit; a less-fresh unit is thus replaced when either unit is purchased, at rate $\lambda_{L,s}+\lambda_o$, or spoils, at rate 2μ, giving a transition from $S_{(L,L)}$ to $S_{(F,L)}$ at rate $\lambda_{L,s}+\lambda_o+2\mu$. The transition rate matrix is

$$Q=\begin{bmatrix} -2\mu & 2\mu & 0 \\ \mu+(1-\psi)\lambda_o & -(2\mu+(1-\psi)\lambda_o) & \mu \\ 0 & \lambda_{L,s}+\lambda_o+2\mu & -(\lambda_{L,s}+\lambda_o+2\mu) \end{bmatrix}. \tag{21}$$

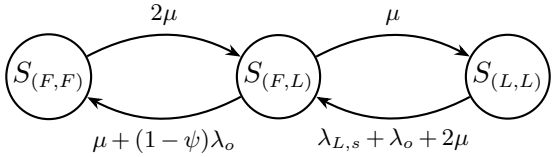


Figure 4 Three-state CTMC for the joint freshness of two units of the same product sharing the shelf under the omnichannel model.

Solving for the steady-state distribution with the normalization $\theta_{(F,F)}+\theta_{(F,L)}+\theta_{(L,L)}=1$ yields

$$\begin{aligned} \theta_{(F,F)} &= \frac{[2\mu+2(1-\psi)\lambda_o]\,(\lambda_{L,s}+\lambda_o+2\mu)}{[6\mu+2(1-\psi)\lambda_o]\,(\lambda_{L,s}+\lambda_o+2\mu)+4\mu^2}, \\ \theta_{(F,L)} &= \frac{4\mu(\lambda_{L,s}+\lambda_o+2\mu)}{[6\mu+2(1-\psi)\lambda_o]\,(\lambda_{L,s}+\lambda_o+2\mu)+4\mu^2}, \\ \theta_{(L,L)} &= \frac{4\mu^2}{[6\mu+2(1-\psi)\lambda_o]\,(\lambda_{L,s}+\lambda_o+2\mu)+4\mu^2}. \end{aligned} \tag{22}$$

To express the grocer's profit, we compute the sales and store-waste rates. In states $S_{(F,F)}$ and $S_{(F,L)}$, where a fresh unit is on the shelf, purchases occur at the combined rate $\lambda_{F,s}+\lambda_o$—an in-store shopper takes the fresh unit and a BOPS shopper is fulfilled with a random one—whereas in state $S_{(L,L)}$ only less-fresh units remain and purchases occur at rate $\lambda_{L,s}+\lambda_o$. Store waste occurs same as in the conventional model. Combining these terms, the long-run average profit of the omnichannel model is

$$\begin{aligned} \pi(p) = {} & p\left[(\lambda_{F,s}+\lambda_o)(\theta_{(F,F)}+\theta_{(F,L)})+(\lambda_{L,s}+\lambda_o)\theta_{(L,L)}\right] \\ & -c\left[(\lambda_{F,s}+\lambda_o)(\theta_{(F,F)}+\theta_{(F,L)})+(\lambda_{L,s}+\lambda_o)\theta_{(L,L)}+\mu\theta_{(F,L)}+2\mu\theta_{(L,L)}\right]. \end{aligned} \tag{23}$$

The grocer's profit-maximization problem is

$$\pi^B=\max_{p\geq 0}\ \pi(p) \quad \text{s.t. Equations } (18),(22)-(23),(3)-(4), \text{ and } (9)-(10).$$

Our main results remain valide in this model extension, the detailed analysis of which is provided in Online Appendix A.2.

6.2. Non-Immediate Restocking

In this section, we relax the assumption that the grocer restocks instantaneously. When a unit leaves the shelf—through a sale or spoilage—the grocer replenishes it after a random delay, modeling the time to source, transport, and shelve a replacement. We assume the replenishment time is exponentially distributed with rate σ, so that $1/\sigma$ is the mean restocking lead time; the main model corresponds to the limiting case $\sigma \to \infty$. Because replenishment now takes time, the shelf can sit empty while the grocer awaits a fresh unit, introducing stockouts and lost sales as a new channel through which restocking speed shapes both waste and profit—an effect that is absent when restocking is instantaneous.

Retail operations. With non-immediate restocking, the shelf may be empty while the grocer awaits replenishment, so we augment the state space with an empty state. The freshness of the on-shelf product is captured by a CTMC with state space $\{S_F, S_L, S_\emptyset\}$, where S_F and S_L denote a fresh and a less-fresh unit on the shelf, respectively, and $S_\emptyset$ denotes an empty shelf awaiting restock. The fresh unit either is purchased—at the fresh demand rate λ_F—or deteriorates to less-fresh at rate μ; in the former case the shelf empties, so the system transitions from S_F to $S_\emptyset$ at rate λ_F and from S_F to S_L at rate μ. From S_L, the less-fresh unit leaves the shelf either through a customer purchase (effective rate λ_L) or through in-store spoilage (rate μ); unlike the baseline, the shelf is not refilled instantly but instead becomes empty, so the system transitions to $S_\emptyset$ at the combined rate $\lambda_L + \mu$. From $S_\emptyset$, the grocer replenishes a fresh unit at rate σ, returning the system to S_F. Figure 5 presents the rate diagram that captures the transition of the product's state at store. The transition rate matrix for this CTMC is

$$Q = \begin{bmatrix} -(\lambda_F + \mu) & \mu & \lambda_F \\ 0 & -(\lambda_L + \mu) & \lambda_L + \mu \\ \sigma & 0 & -\sigma \end{bmatrix}.$$

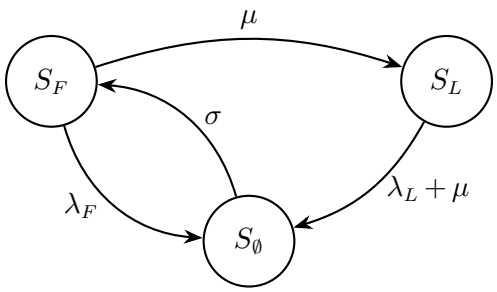


Figure 5 Three-state CTMC for a single product under non-immediate restocking.

Let θ_F, θ_L, and $\theta_\emptyset$ denote the steady-state probabilities of the three shelf states. The balance equations together with $\theta_F + \theta_L + \theta_\emptyset = 1$ yield

$$\begin{aligned}\theta_F &= \frac{(\lambda_L+\mu)\sigma}{(\lambda_L+\mu)\sigma + \mu\sigma + (\lambda_F+\mu)(\lambda_L+\mu)},\\ \theta_L &= \frac{\mu\sigma}{(\lambda_L+\mu)\sigma + \mu\sigma + (\lambda_F+\mu)(\lambda_L+\mu)},\\ \theta_\emptyset &= \frac{(\lambda_F+\mu)(\lambda_L+\mu)}{(\lambda_L+\mu)\sigma + \mu\sigma + (\lambda_F+\mu)(\lambda_L+\mu)}.\end{aligned} \tag{24}$$

As $\sigma \to \infty$, $\theta_\emptyset \to 0$ and (θ_F, θ_L) reduce to the steady state in Equation (1). The empty state introduces a new feature: a customer who arrives to an empty shelf finds no product to purchase, so demand is lost; only customers who arrive in states S_F or S_L can buy. Given the product price p, the grocer's long-run average profit function retains the form of Equation (2).

Customer behavior and utilities. The household-side dynamics, the expected valuations V_F and V_L in Equations (3)–(4), and the hassle-cost parameterization in Equation (8) are unchanged. An in-store customer observes the shelf state on arrival and purchases according to the freshness she finds, buying a fresh unit in S_F (if $V_F \geq p$), a less-fresh unit in S_L (if $V_L \geq p$), and nothing in $S_\emptyset$. Her utility remains the same as in Equation (5), with the empty state contributing nothing to her utility.

For the BOPS channel, a customer commits online before fulfillment. If the picker finds the shelf empty at fulfillment, the order cannot be filled and the customer is not charged for it, so she incurs a payment only when a unit is available. This reflects standard omnichannel grocery practice: major grocers charge the customer's payment card at pickup rather than at order placement, and unavailable items for which no substitution is accepted are simply dropped from the order and not charged[2]. Her expected BOPS utility is therefore taken over the non-empty shelf states:

$$u_b = -h_b + \theta_F V_F + \theta_L V_L - (\theta_F + \theta_L)\, p, \tag{25}$$

where $(\theta_F + \theta_L)\, p$ reflects that she pays only when an order is successfully fulfilled.

As in our main model, customers choose the channel yielding the higher non-negative utility, and the realized demand rates are determined in equilibrium by which customers buy and through which channel. The grocer's profit-maximization problem for the conventional model and the omnichannel model is

$$\begin{aligned}\pi^C &= \max_{p\geq 0}\ \pi(p) \quad \text{s.t. Equations (24) and (2) – (6);}\\ \pi^B &= \max_{p\geq 0}\ \pi(p) \quad \text{s.t. Equations (24), (2) – (4), (25), and (6),}\end{aligned}$$

[2] For example, under Kroger's Pickup & Delivery terms, the customer's payment card is charged "the amount of your order upon pickup", and if substitutions are declined the unavailable product "will not be included in your order", with any rejected items credited back to the payment card (The Kroger Co. 2026).

respectively. Our main insights continue to hold when restocking does not happen immediately. The detailed analysis is included in Online Appendix A.3.

6.3. Heterogeneous Consumer Hassle Cost

In the main model, all customers share a common in-store hassle cost h_s. In this section, we relax this assumption and allow the in-store hassle cost to vary across customers. Specifically, we assume each arriving customer draws an in-store hassle cost h_s independently from a with cumulative distribution function $G(\cdot)$ and support on $[\underline{h}, \bar{h}]$, where $0 \le \underline{h} < \bar{h}$. We assume G is continuous and strictly increasing on its support, so that it admits a well-defined inverse. This extension enriches the equilibrium demand structure: rather than all customers participating or balking together and making the same channel choice, customers now sort according to their individual hassle costs, so that in-store shoppers, BOPS shoppers, and non-participants may coexist in equilibrium and the grocer faces a continuous, price-sensitive demand response in place of the main model's all-or-nothing demand.

6.3.1. Conventional Model A customer with hassle cost h_s visits the store if and only if $u_s(h_s) = -h_s + \theta_F(V_F - p)^+ + \theta_L(V_L - p)^+ \ge 0$. Because $u_s(h_s)$ is strictly decreasing in h_s, for a given price p, a customer commits an in-store purchase when her hassle cost falls below the threshold

$$A(p) \triangleq \theta_F(V_F - p)^+ + \theta_L(V_L - p)^+. \tag{26}$$

We let $\Phi(p)$ be the fraction of arriving customers who visit the store. That is,

$$\Phi(p) \triangleq \Pr\{h_s \le A(p)\} = G\big(A(p)\big). \tag{27}$$

In contrast to the main model's indicator demand, $\Phi(p)$ is continuous in the price p as the grocer raises the price, the threshold $A(p)$ falls and customers drop out gradually rather than all at once. The realized demand rates for fresh and less-fresh units become

$$\lambda_F = \Lambda\,\Phi(p)\,\mathbf{1}\{V_F - p \ge 0\} \quad \text{and} \quad \lambda_L = \Lambda\,\Phi(p)\,\mathbf{1}\{V_L - p \ge 0\}. \tag{28}$$

The steady-state freshness composition (θ_F, θ_L) in Equation (1) and the profit function $\pi(p)$ in Equation (2) retain their form, now evaluated at the continuous demand rates in Equation (28). Note that $A(p)$ itself depends on (θ_F, θ_L), which in turn depend on λ_L; each equilibrium is a fixed point at which the participation fraction $\Phi(p)$ and the steady-state composition are mutually consistent. The grocer's profit-maximization problem retains the form

$$\pi^C = \max_{p \ge 0} \pi(p) \quad \text{s.t. Equations } (1) - (4) \text{ and } (26) - (28).$$

6.3.2. Omnichannel Model When BOPS is available, the heterogeneity carries through to the channel-choice as well. A customer with in-store hassle h_s has BOPS hassle $h_b = \alpha h_s + h_o$, where $\alpha \in (0,1)$, as described in Equation (8). She shops in store when $u_s(h_s) > u_b(h_s)$ and $u_s(h_s) \geq 0$, orders through BOPS when $u_b(h_s) \geq \max\{u_s(h_s), 0\}$, and balks otherwise, where u_s and u_b are given in Equations (5) and (7) evaluated at the customer's own h_s.

Since $u_s(h_s) - u_b(h_s) = h_o - (1-\alpha)\, h_s + \theta_L\, (p - V_L)^+$ is strictly decreasing in h_s, the comparison between $u_s(h_s)$ and $u_b(h_s)$ depends on a threshold in h_s:

$$B(p) \triangleq \frac{1}{1-\alpha}\left[h_o + \theta_L\,(p - V_L)^+\right]. \tag{29}$$

That is, $u_s(h_s) > u_b(h_s)$ if and only if $h_s < B(p)$. Recall from the conventional model that $u_s(h_s) \geq 0$ if and only if $h_s \leq A(p)$. A customer therefore shops in store when $h_s \leq A(p)$ and $h_s < B(p)$.

In addition, $u_b(h_s) \geq 0$ depends on a threshold in h_s:

$$C(p) \triangleq \frac{1}{\alpha}\left[\theta_F V_F + \theta_L V_L - p - h_o\right], \tag{30}$$

that is, $u_b(h_s) \geq 0$ if and only if $h_s \leq C(p)$. A customer therefore shops through BOPS when $B(p) \leq h_s \leq C(p)$.

Combining the two cases, customers shop in store when $h_s \leq A(p)$ and $h_s < B(p)$, and through BOPS when $B(p) \leq h_s \leq C(p)$. Since G is continuous, the channel-indifference point has zero probability. Thus, the in-store fraction is

$$\Phi_s(p) \triangleq \Pr\{h_s \leq \min\{A(p), B(p)\}\} = G\big(\min\{A(p), B(p)\}\big), \tag{31}$$

and the BOPS fraction is the mass of customers with $B(p) \leq h_s \leq C(p)$,

$$\Phi_b(p) \triangleq \big(G(C(p)) - G(B(p))\big)^+. \tag{32}$$

Hence, the realized demand rates are

$$\lambda_F = \Lambda\left[\Phi_s(p)\,\mathbf{1}\{V_F - p \geq 0\} + \Phi_b(p)\right] \quad \text{and} \quad \lambda_L = \Lambda\left[\Phi_s(p)\,\mathbf{1}\{V_L - p \geq 0\} + \Phi_b(p)\right]. \tag{33}$$

Similar to the conventional model, (θ_F, θ_L) in Equation (1) and the profit function $\pi(p)$ retain their form, now evaluated at the demand rates in Equation (33), and each equilibrium is a fixed point at which the participation fractions $\Phi_s(p)$ and $\Phi_b(p)$ and the steady-state composition are mutually consistent. The grocer's profit-maximization problem retains the form

$$\pi^B = \max_{p \geq 0}\ \pi(p) \quad \text{s.t. Equations } (1)-(4) \text{ and } (29)-(33).$$

Our main insights continue to hold under heterogeneous hassle costs. The detailed analysis is included in Online Appendix A.4.

7. Concluding Remarks

This paper studies a grocer selling a perishable product through in-store shopping and BOPS. In-store customers observe freshness before deciding whether to buy, while BOPS customers commit online without this information. We use two coupled continuous-time Markov chains (CTMCs) to track product freshness on the shelf and consumption or spoilage after purchase. The grocer sets a common price across channels to maximize long-run average profit, and customers choose whether to purchase and which channel to use. Within this framework, we examine how BOPS changes the profit and waste effects of freshness technology and a per-unit tax on retail waste.

We find that slowing store spoilage weakly increases optimal gross profit before technology costs, but its effect on store waste depends critically on the retail context. When in-store shopping is convenient, freshness technology reduces store waste in the conventional single-channel setting but can increase it in an omnichannel environment with BOPS; when in-store shopping is inconvenient, this pattern can reverse. Grocers should therefore evaluate adoption by weighing the implementation cost against the channel- and density-specific profit gain and by accounting for how improved freshness changes customers' channel choices. A retail waste tax creates a similarly context-dependent trade-off. Although the tax weakly reduces store waste, it weakly increases household waste and can therefore raise total waste when household spoilage is sufficiently high relative to consumption. BOPS can either mitigate or amplify this unintended effect: when online shopping is relatively convenient, the tax-induced increase in total waste can be smaller under BOPS than in the conventional setting, whereas this comparison can reverse when online shopping is relatively inconvenient. Policymakers should therefore evaluate waste taxes together with the pricing and purchasing responses they induce and assess store and household waste jointly, because lower retail disposal may reflect a shift of waste to households rather than a genuine reduction in total waste.

To isolate how channel structure shapes customer behavior and waste outcomes, our analysis abstracts from endogenous inventory decisions. Our extension with non-immediate restocking allows stockouts and shows that the main channel mechanisms remain relevant when products are not always available, although stocking and replenishment decisions remain exogenous. A natural direction for future research is to endogenize these decisions and examine how inventory policies interact with pricing and channel choice to shape store waste, household waste, and lost sales. Another promising direction is to introduce competition among grocers. Research on supplier auditing shows that downstream competition can discourage collectively beneficial actions (Chen et al. 2020), motivating an examination of how competition affects grocers' pricing, channel offerings, and waste outcomes.

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Online Appendix to "Food Waste in Omnichannel Grocery Retail: The Role of Buy Online and Pick Up In Store"

A.1. Proofs of Results in the Main Model

In this section, we provide the proofs of the results in the main model, which include Lemmas 1 and 2 and Propositions 1–4.

Proof of Lemma 1. We consider two regimes: (*i*) customers purchase only fresh products (i.e., $V_L < p \le V_F$), and (*ii*) customers purchase both fresh and less-fresh products (i.e., $p \le V_L$).

(i) $V_L < p \le V_F$: When customers purchase only fresh products, the price satisfies $V_L < p \le V_F$, and the demand rates are $\lambda_F = \Lambda$ and $\lambda_L = 0$. The resulting steady state is $\theta_F = \theta_L = \frac{1}{2}$. Substituting into u_s gives $u_s = -h_s + \frac{1}{2}(V_F - p)$, so a customer participates ($u_s \ge 0$) if and only if $p \le V_F - 2h_s$.

The profit function in Equation (2) becomes $\pi(p) = \frac{\Lambda}{2}(p-c) - \frac{c\mu}{2}$, which is strictly increasing in p. The optimal price is therefore $p^C = \min\{V_F, V_F - 2h_s\} = V_F - 2h_s$, with optimal profit $\pi^C = \pi(p^C) = \frac{\Lambda}{2}(V_F - 2h_s - c) - \frac{c\mu}{2}$. This regime is internally consistent (i.e., $p^C > V_L$) if and only if $h_s < \frac{V_F - V_L}{2}$.

(ii) $p \le V_L$: When customers purchase both fresh and less-fresh products, the price satisfies $p \le V_L$, and the demand rates are $\lambda_F = \lambda_L = \Lambda$. The resulting steady state is $\theta_F = \frac{\Lambda+\mu}{\Lambda+2\mu}$ and $\theta_L = \frac{\mu}{\Lambda+2\mu}$. Substituting into u_s, and using $\theta_F + \theta_L = 1$,

$$u_s = -h_s + \frac{\Lambda+\mu}{\Lambda+2\mu}(V_F - p) + \frac{\mu}{\Lambda+2\mu}(V_L - p) = -h_s + \frac{(\Lambda+\mu)V_F + \mu V_L}{\Lambda+2\mu} - p,$$

so a customer participates ($u_s \ge 0$) if and only if $p \le \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_s$.

The profit function becomes $\pi(p) = \Lambda(p-c) - \frac{c\mu^2}{\Lambda+2\mu}$, which is strictly increasing in p. The grocer therefore charges the highest price consistent with both less-fresh acceptance ($p \le V_L$) and participation ($p \le \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_s$), giving $p^C = \min\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_s\}$ and optimal profit $\pi^C = \Lambda(p^C - c) - \frac{c\mu^2}{\Lambda+2\mu}$. Specifically,

- when $h_s \le \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - V_L = \frac{(\Lambda+\mu)(V_F-V_L)}{\Lambda+2\mu}$, the acceptance bound binds, i.e., $p^C = V_L$ and $\pi^C = \Lambda(V_L - c) - \frac{c\mu^2}{\Lambda+2\mu}$;
- when $h_s > \frac{(\Lambda+\mu)(V_F-V_L)}{\Lambda+2\mu}$, the participation bound binds, i.e., $p^C = \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_s$ and $\pi^C = \Lambda(\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_s - c) - \frac{c\mu^2}{\Lambda+2\mu}$.

Note that $p^C \ge 0$ if and only if $h_s \le \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$, which provides an upper bound on h_s.

We then check the upper bound of h_s from regimes (i) and (ii). Note that $\frac{V_F-V_L}{2} < \frac{(\Lambda+\mu)(V_F-V_L)}{\Lambda+2\mu}$ as $\frac{\Lambda+\mu}{\Lambda+2\mu} > \frac{1}{2}$ for $\Lambda > 0$ and $\frac{(\Lambda+\mu)(V_F-V_L)}{\Lambda+2\mu} < \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$. Therefore, when $h_s < \frac{V_F-V_L}{2}$, setting the optimal price within both regimes (i) and (ii) is feasible; when $\frac{V_F-V_L}{2} \le h_s \le \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$, setting

the optimal price only in regime (ii) is feasible. Thus, we need to further decide the optimal price when $h_s < \frac{V_F - V_L}{2}$. Let π_1^C and π_2^C denote the optimal profits under regime (i) and (ii), respectively:

$$\pi_1^C = \frac{\Lambda}{2}(V_F - 2h_s - c) - \frac{c\mu}{2}, \qquad \pi_2^C = \Lambda(V_L - c) - \frac{c\mu^2}{\Lambda + 2\mu}.$$

The grocer chooses regime (i) over regime (ii) if and only if $\pi_1^C \geq \pi_2^C$, i.e.,

$$\frac{\Lambda}{2}(V_F - 2h_s - c) - \frac{c\mu}{2} \geq \Lambda(V_L - c) - \frac{c\mu^2}{\Lambda + 2\mu},$$

which is equivalent to

$$h_s \leq \frac{V_F}{2} - V_L + \frac{c(\Lambda+\mu)}{2(\Lambda+2\mu)} = \frac{V_F - 2V_L}{2} + \frac{c(\Lambda+\mu)}{2(\Lambda+2\mu)}.$$

When the two feasible regimes yield the same maximal profit, we select regime (i), consistently with Lemma 1. Thus, the grocer sets the optimal price as $p^C = V_F - 2h_s$ when $h_s \leq \min\{\frac{V_F-2V_L}{2} + \frac{c(\Lambda+\mu)}{2(\Lambda+2\mu)}, \frac{V_F-V_L}{2}\}$ and as $p^C = \min\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_s\}$ when $\min\{\frac{V_F-2V_L}{2} + \frac{c(\Lambda+\mu)}{2(\Lambda+2\mu)}, \frac{V_F-V_L}{2}\} < h_s \leq \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$.

Finally, we ensure that the resulting profit is positive. That is,

- when $V_L < p \leq V_F$: $\pi^C > 0$ if and only if $c < c_1(h_s) \triangleq \frac{\Lambda(V_F-2h_s)}{\Lambda+\mu}$.
- when $p \leq V_L$: $\pi^C > 0$ if and only if $c < c_2(h_s) \triangleq \frac{\Lambda(\Lambda+2\mu)}{(\Lambda+\mu)^2}p^C$, where $p^C = \min\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_s\}$.

Since the grocer selects the more profitable regime of the feasible regimes, π^C equals the larger of the two regime profits, so $\pi^C > 0$ if and only if at least one *feasible* regime earns a positive profit. Regime (i) is feasible only for $h_s < \frac{V_F-V_L}{2}$, whereas the regime (ii) is feasible for $h_s \leq \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$. Combining the two cases:

- When $h_s < \frac{V_F-V_L}{2}$, both regimes are feasible, so $\pi^C > 0$ if and only if $c < c_1(h_s)$ or $c < c_2(h_s)$, i.e., $c < \max\{c_1(h_s), c_2(h_s)\}$.
- When $\frac{V_F-V_L}{2} \leq h_s \leq \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$, only the regime (ii) is feasible, so $\pi^C > 0$ if and only if $c < c_2(h_s)$.

  We further show that $c_1(h_s) \leq c_2(h_s)$ under this case. Note that $c_1(h_s) \leq c_2(h_s)$ is equivalent to $(V_F - 2h_s)(\Lambda + \mu) \leq (\Lambda + 2\mu)\,p^C$. When $p^C = V_L$, this holds because $V_F - 2h_s \leq V_L$ and $\Lambda + \mu < \Lambda + 2\mu$; when $p^C = \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_s$, this holds because $(\Lambda+2\mu)\big(\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_s\big) - (\Lambda+\mu)(V_F - 2h_s) = \mu V_L + \Lambda h_s > 0$.

  Hence, $c < c_2(h_s)$ coincides with $c < \max\{c_1(h_s), c_2(h_s)\}$.

In addition, for $h_s > \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$, no non-negative price is feasible and $\pi^C = 0$. Therefore, the grocer's profit is positive if and only if

$$h_s \leq \frac{(\Lambda+\mu)V_F + \mu V_L}{\Lambda + 2\mu} \text{ and } c < \max\big\{c_1(h_s), c_2(h_s)\big\}.$$

Combining the discussion above, under the condition $h_s \le \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$ and $c < \max\{c_1(h_s), c_2(h_s)\}$ (to ensure the grocer earn positive profit), the grocer sets its optimal price as $p^C = V_F - 2h_s$ when $h_s \le \min\{\frac{V_F-2V_L}{2} + \frac{c(\Lambda+\mu)}{2(\Lambda+2\mu)}, \frac{V_F-V_L}{2}\}$ and as $p^C = \min\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_s\}$ when $\min\{\frac{V_F-2V_L}{2} + \frac{c(\Lambda+\mu)}{2(\Lambda+2\mu)}, \frac{V_F-V_L}{2}\} < h_s \le \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$. The optimal profit is calculated accordingly following the profit function in Equation (2). This completes the proof. □

Proof of Lemma 2. Write

$$\bar{V} = \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}, \text{ and } \mathcal{H}_s = \min\left\{\frac{V_F-V_L}{2}, \frac{V_F-2V_L}{2} + \frac{c(\Lambda+\mu)}{2(\Lambda+2\mu)}\right\},$$

where $\mathcal{H}_s$ is the conventional pricing-regime threshold in Lemma 1.

Under BOPS adoption, customers commit online and accept the product regardless of its freshness at pickup, so the demand rates are $\lambda_F = \lambda_L = \Lambda$ and the steady state is $\theta_F = \frac{\Lambda+\mu}{\Lambda+2\mu}$ and $\theta_L = \frac{\mu}{\Lambda+2\mu}$. Therefore, the BOPS utility is $u_b = -h_b + \theta_F V_F + \theta_L V_L - p = -h_b + \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - p$. Recall from Equation (2) that the profit function is $\pi(p) = \Lambda(p-c) - \frac{c\mu^2}{\Lambda+2\mu}$, which is strictly increasing in p. The grocer therefore charges the highest price at which every customer prefers BOPS, i.e., $u_b \ge 0$ and $u_b \ge u_s$. We discuss the two constraints separately.

- For $u_b \ge 0$, we have $p \le p_1 \triangleq \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_b$.
- For $u_b \ge u_s$, we consider two sub-cases according to whether an in-store customer accepts a less-fresh product or not.
  (i) $V_L < p \le V_F$: only the fresh product is accepted in-store, so $u_s = -h_s + \theta_F(V_F - p)$. Using $1-\theta_F = \theta_L$, we have $u_b - u_s = \theta_L(V_L - p) + (h_s - h_b)$. The condition $u_b \ge u_s$ is equivalent to $\theta_L(p - V_L) \le h_s - h_b$, i.e.,

  $$p \le p_2 \triangleq V_L + \frac{\Lambda+2\mu}{\mu}(h_s - h_b).$$

  This bound exceeds V_L if and only if $h_b < h_s$, i.e., $h_o < (1-\alpha)h_s$.
  (ii) $p \le V_L$: both fresh and less-fresh products are accepted in-store, so $u_s = -h_s + \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - p$ and $u_b - u_s = h_s - h_b$. Thus, $u_b \ge u_s$ holds automatically whenever $h_b \le h_s$, i.e., $h_o \le (1-\alpha)h_s$; it imposes no additional bound on p as long as $h_o \le (1-\alpha)h_s$.

Therefore, the feasible price set is $p \le p_1$ and $p \le p_2$ under the condition $h_o \le (1-\alpha)h_s$. Then, the highest price inducing BOPS usage and its associated profit are

$$\widehat{p}^B = \min\{p_1, p_2\} = \min\left\{\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_b,\ V_L + \frac{\Lambda+2\mu}{\mu}(h_s - h_b)\right\},$$

with associated profit $\widehat{\pi}^B = \Lambda(\widehat{p}^B - c) - \frac{c\mu^2}{\Lambda+2\mu}$. Note that both p_1 and p_2 are strictly decreasing in h_o, so $\widehat{p}^B$ and $\widehat{\pi}^B$ are continuous and strictly decreasing in h_o.

We compare this candidate with the conventional optimum using the two cases in Lemma 1.

Case (i): $0 < h_s \le \mathcal{H}_s$. The conventional optimum sells only fresh products when $h_s \le \mathcal{H}_s$, with

$$p^C = V_F - 2h_s, \qquad \pi^C = \frac{\Lambda}{2}(V_F - 2h_s - c) - \frac{c\mu}{2}.$$

The equality $\widehat{\pi}^B = \pi^C$ corresponds to the price

$$p_3 \triangleq \frac{V_F - 2h_s + c}{2} - \frac{c\mu}{2(\Lambda + 2\mu)}.$$

Hence, $\widehat{\pi}^B > \pi^C$ if and only if $\widehat{p}^B > p_3$, or equivalently, both $p_1 > p_3$ and $p_2 > p_3$, giving two upper bounds on h_o:

- $p_1 > p_3$ is equivalent to

$$h_o < h_{o1} \triangleq \frac{(\Lambda + \mu)V_F + \mu V_L}{\Lambda + 2\mu} - \alpha h_s - \frac{V_F - 2h_s + c}{2} + \frac{c\mu}{2(\Lambda + 2\mu)};$$

- $p_2 > p_3$ is equivalent to

$$h_o < h_{o2} \triangleq \frac{\mu}{\Lambda + 2\mu}\left(V_L - \frac{V_F - 2h_s + c}{2} + \frac{c\mu}{2(\Lambda + 2\mu)}\right) + (1 - \alpha)h_s.$$

Thus, $\widehat{\pi}^B > \pi^C$ if and only if $h_o < \min\{h_{o1}, h_{o2}\}$. Moreover, $h_s \le \mathcal{H}_s$ implies

$$p_3 - V_L = \frac{V_F - 2V_L}{2} + \frac{c(\Lambda + \mu)}{2(\Lambda + 2\mu)} - h_s \ge 0 \Rightarrow \min\{h_{o1}, h_{o2}\} \le h_{o2} \le (1 - \alpha)h_s.$$

Case (ii): $\mathcal{H}_s < h_s \le \bar{V}$. The conventional optimum sells both freshness levels, with

$$p^C = \min\{V_L, \bar{V} - h_s\}, \qquad \pi^C = \Lambda(p^C - c) - \frac{c\mu^2}{\Lambda + 2\mu}.$$

For every $h_o < (1 - \alpha)h_s$, we have $h_b < h_s$, and hence $p_1 > \bar{V} - h_s$ and $p_2 > V_L$. Therefore,

$$\widehat{p}^B = \min\{p_1, p_2\} > \min\{\bar{V} - h_s, V_L\} = p^C \Rightarrow \widehat{\pi}^B - \pi^C = \Lambda(\widehat{p}^B - p^C) > 0.$$

BOPS is thus strictly profitable in that range, and its adoption threshold is $(1 - \alpha)h_s$.

Combining the two cases, define

$$\mathcal{H}_o = \begin{cases} \min\{h_{o1}, h_{o2}\}, & 0 < h_s \le \mathcal{H}_s, \\ (1 - \alpha)h_s, & \mathcal{H}_s < h_s \le \bar{V}. \end{cases}$$

Then $\mathcal{H}_o \le (1 - \alpha)h_s$, and $\widehat{\pi}^B > \pi^C$ if and only if $h_o < \mathcal{H}_o$.

To complete the comparison, any equilibrium without BOPS usage earns at most π^C. Thus, when $h_o < \mathcal{H}_o$, the optimal outcome is $p^B = \widehat{p}^B$ and $\pi^B = \widehat{\pi}^B > \pi^C$.

When $\mathcal{H}_o \le h_o < (1 - \alpha)h_s$, only case (i) can apply, and $\widehat{\pi}^B \le \pi^C$. We verify that the conventional fresh-only outcome remains feasible. At $p^C = V_F - 2h_s$ and $\theta_F = \theta_L = 1/2$, the in-store utility is zero. If a customer weakly preferred BOPS at this price, then $h_s - h_b \ge \frac{p^C - V_L}{2}$. Under the BOPS

steady state, the shelf is fresher and $\theta_L = \mu/(\Lambda + 2\mu) < 1/2$, so both BOPS participation and channel-choice constraints would also hold at p^C. Serving all customers online at that price would yield

$$\Lambda(p^C - c) - \frac{c\mu^2}{\Lambda + 2\mu} > \frac{\Lambda}{2}(p^C - c) - \frac{c\mu}{2} = \pi^C,$$

where $p^C > c$ follows from the maintained assumption $\pi^C > 0$. This contradicts $\widehat{\pi}^B \le \pi^C$. Hence, the conventional fresh-only outcome is feasible, and the grocer selects $p^B = V_F - 2h_s$ with no BOPS usage and $\pi^B = \pi^C$, selecting this outcome when profits tie. This completes the proof. □

Proof of Proposition 1. We prove the claim for the conventional model ($\pi^C_{\mu'} \ge \pi^C_{\mu}$) and the omnichannel model ($\pi^B_{\mu'} \ge \pi^B_{\mu}$) separately.

We first record two monotonicity properties. For a fixed demand regime λ_L, the steady-state composition$\theta_F(\mu) = \frac{\lambda_L+\mu}{\lambda_L+2\mu}$ and $\theta_L(\mu) = \frac{\mu}{\lambda_L+2\mu}$ satisfies $\frac{\partial \theta_L(\mu)}{\partial \mu} = \frac{\lambda_L}{(\lambda_L+2\mu)^2} \ge 0$, so $\theta_L(\mu)$ is non-decreasing in μ and $\theta_F(\mu) = 1 - \theta_L(\mu)$ is correspondingly non-increasing in μ. Since $\mu' < \mu$, we therefore have $\theta_F(\mu') \ge \theta_F(\mu)$ and $\theta_L(\mu') \le \theta_L(\mu)$. Similarly, the store-waste rate $W_s(\mu) = \mu\theta_L(\mu) = \frac{\mu^2}{\lambda_L+2\mu}$ is strictly increasing in μ for fixed λ_L, since $\frac{\partial W_s(\mu)}{\partial \mu} = \frac{2\mu(\lambda_L+\mu)}{(\lambda_L+2\mu)^2} > 0$; hence $W_s(\mu') < W_s(\mu)$.

Conventional model. Let p^C_μ denote the optimal price when the spoilage rate is μ. We show that p^C_μ is feasible under μ' and yields a weakly higher profit.

To show that p^C_μ is feasible under μ', we show that every customer still participates ($u_s \ge 0$) at price p^C_μ under μ'. A customer's in-store utility at price p and spoilage rate μ is

$$u_s(p;\mu) = -h_s + \theta_F(\mu)(V_F - p)^+ + \theta_L(\mu)(V_L - p)^+.$$

When $p \le V_L$, by $\theta_F(\mu) + \theta_L(\mu) = 1$, we have $u_s(p;\mu) = -h_s + V_L + \theta_F(\mu)(V_F - V_L) - p$, which is non-decreasing as μ falls because $\theta_F(\mu') \ge \theta_F(\mu)$ and $V_F \ge V_L$; hence $u_s(p;\mu') \ge u_s(p;\mu)$. When $V_L < p \le V_F$, $u_s(p;\mu) = -h_s + \theta_F(\mu)(V_F - p)$ with $V_F - p \ge 0$, so again $u_s(p;\mu') \ge u_s(p;\mu)$. In both cases $u_s(p^C_\mu;\mu') \ge u_s(p^C_\mu;\mu) \ge 0$, so p^C_μ remains feasible under μ'.

Because participation holds under both spoilage rates, and because whether a customer accepts a less-fresh unit is determined solely by comparing the fixed price p^C_μ to V_L, the induced demand rates λ_F and λ_L are the same under μ and μ'. The profit at price p^C_μ under μ' is

$$\pi_{\mu'}(p^C_\mu) = (p^C_\mu - c)\big(\theta_F(\mu')\lambda_F + \theta_L(\mu')\lambda_L\big) - c\,W_s(\mu'),$$

and, since $\pi^C_\mu = \pi_\mu(p^C_\mu)$, we have

$$\pi_{\mu'}(p^C_\mu) - \pi^C_\mu = (p^C_\mu - c)\big[\lambda_F\big(\theta_F(\mu') - \theta_F(\mu)\big) + \lambda_L\big(\theta_L(\mu') - \theta_L(\mu)\big)\big] - c\big(W_s(\mu') - W_s(\mu)\big).$$

We consider the two pricing regimes.

(i) $V_L < p_\mu^C \le V_F$: Here $\lambda_F = \Lambda$, $\lambda_L = 0$, and $p_\mu^C = V_F - 2h_s$. With $\lambda_L = 0$ we have $\theta_F(\mu) = \theta_L(\mu) = \frac{1}{2}$ for every μ, so the sales term vanishes and $W_s(\mu) = \frac{\mu}{2}$. Hence,

$$\pi_{\mu'}(p_\mu^C) - \pi_\mu^C = -c\Big(\tfrac{\mu'}{2} - \tfrac{\mu}{2}\Big) = \tfrac{c}{2}(\mu - \mu') \ge 0.$$

(ii) $p_\mu^C \le V_L$: Here $\lambda_F = \lambda_L = \Lambda$. Using $\theta_F + \theta_L = 1$, the sales term is $(p_\mu^C - c)\Lambda\big[(\theta_F(\mu') + \theta_L(\mu')) - (\theta_F(\mu) + \theta_L(\mu))\big] = (p_\mu^C - c)\Lambda(1-1) = 0$, and $W_s(\mu) = \frac{\mu^2}{\Lambda + 2\mu}$. Hence,

$$\pi_{\mu'}(p_\mu^C) - \pi_\mu^C = -c\Big(\tfrac{(\mu')^2}{\Lambda+2\mu'} - \tfrac{\mu^2}{\Lambda+2\mu}\Big) \ge 0,$$

where the inequality holds because $c \ge 0$ and $W_s(\mu)$ is strictly increasing in μ.

In both regimes $\pi_{\mu'}^C \ge \pi_{\mu'}(p_\mu^C) \ge \pi_\mu^C$, so $\pi_{\mu'}^C \ge \pi_\mu^C$.

Omnichannel model. If no customer uses BOPS at μ, customers purchase only fresh products in store. Retain the original optimal price p_μ^B under μ'. Under this demand regime, $\theta_F = \theta_L = 1/2$ at both spoilage rates, so the participation and channel-choice inequalities are unchanged. Hence, the same outcome remains feasible and

$$\pi_{\mu'}(p_\mu^B) - \pi_\mu^B = \frac{c}{2}(\mu - \mu') \ge 0.$$

It follows that $\pi_{\mu'}^B \ge \pi_{\mu'}(p_\mu^B) \ge \pi_\mu^B$. Otherwise BOPS is adopted at μ, all customers commit online and purchase regardless of freshness, so $\lambda_F = \lambda_L = \Lambda$ and the store-waste rate is $W_s(\mu) = \frac{\mu^2}{\Lambda+2\mu}$ in every pricing regime. Let p_μ^B denote the optimal price under μ. We show that p_μ^B is feasible under μ' and yields a weakly higher profit.

Price p is feasible under μ' if and only if $u_b(p;\mu') \ge 0$ and $u_b(p;\mu') \ge u_s(p;\mu')$. For the constraint $u_b(p;\mu') \ge 0$, we have

$$u_b(p;\mu') = -h_b + \theta_F(\mu')V_F + \theta_L(\mu')V_L - p \ge -h_b + \theta_F(\mu)V_F + \theta_L(\mu)V_L - p = u_b(p;\mu) \ge 0,$$

where the inequality uses $\theta_F(\mu') \ge \theta_F(\mu)$, $\theta_L(\mu') \le \theta_L(\mu)$, and $V_F \ge V_L$. For the constraint $u_b(p;\mu') \ge u_s(p;\mu')$, we know that

$$u_b(p;\mu) - u_s(p;\mu) = \begin{cases} h_s - h_b, & p \le V_L, \\ h_s - h_b + \theta_L(\mu)(V_L - p), & V_L < p \le V_F. \end{cases}$$

When $p \le V_L$, this difference does not depend on μ; when $V_L < p \le V_F$, it is non-increasing in $\theta_L(\mu)$ (as $V_L - p < 0$) and hence non-decreasing as μ falls. In either case $u_b(p;\mu') - u_s(p;\mu') \ge u_b(p;\mu) - u_s(p;\mu) \ge 0$, so $u_b(p;\mu') \ge u_s(p;\mu')$ continues to hold. Hence p_μ^B is feasible under μ'.

Finally, because the sales rate $\theta_F\lambda_F + \theta_L\lambda_L = \Lambda(\theta_F + \theta_L) = \Lambda$ and the price p_μ^B are held fixed, only the store-waste cost changes:

$$\pi_{\mu'}(p_\mu^B) - \pi_\mu^B = -c\Big(\tfrac{(\mu')^2}{\Lambda+2\mu'} - \tfrac{\mu^2}{\Lambda+2\mu}\Big) \ge 0.$$

Therefore, $\pi_{\mu'}^B \ge \pi_{\mu'}(p_\mu^B) \ge \pi_\mu^B$, which completes the proof. □

Proof of Proposition 2. Fix $0 < \mu' < \mu$ and set $\mathcal{H}_s = \mathcal{H}_s(\mu) = \min\{\frac{V_F - V_L}{2}, \frac{V_F - 2V_L}{2} + \frac{c(\Lambda+\mu)}{2(\Lambda+2\mu)}\}$. We prove the proposition for the conventional model and the omnichannel model separately.

Conventional model. Note that $\frac{\Lambda+\mu}{\Lambda+2\mu}$ is strictly decreasing in μ, so $\mathcal{H}_s(\mu)$ is non-increasing in μ; hence, $\mathcal{H}_s(\mu') \geq \mathcal{H}_s(\mu)$ whenever $\mu' < \mu$.

By Lemma 1, the conventional grocer sells only fresh products when $h_s \leq \mathcal{H}_s(\mu)$ and both freshness levels when $h_s > \mathcal{H}_s(\mu)$. At $h_s = \mathcal{H}_s(\mu)$, we follow the fresh-only selection in Lemma 1; since $\mathcal{H}_s(\mu') \geq \mathcal{H}_s(\mu)$, store waste then decreases from $\mu/2$ to $\mu'/2$. At this boundary, set $\mathcal{M}_{c1} = \mathcal{M}_{c2} = 2\mu^2/(\Lambda+2\mu)$, so the conventional waste-increase interval is empty. We discuss the two strict cases below.

- $h_s < \mathcal{H}_s(\mu)$: As $\mu > \mu'$, we have $h_s < \mathcal{H}_s(\mu) \leq \mathcal{H}_s(\mu')$.Thus, in-store customers purchase only fresh products at both μ and μ', which is equivalent to $\lambda_F = \Lambda$ and $\lambda_L = 0$. Recall that the store-side waste is $W_s^C(\mu) = \mu\theta_L = \frac{1}{2}\mu$ for a given μ. Therefore, $W_s^C(\mu) = \mu/2 > \mu'/2 = W_s^C(\mu')$.
- $h_s > \mathcal{H}_s(\mu)$: In-store customers purchase both fresh and less fresh products at μ, which is equivalent to $\lambda_F = \lambda_L = \Lambda$, so $W_s^C(\mu) = \mu\theta_L = \frac{\mu^2}{\Lambda+2\mu}$. When the store spoilage rate is reduced to μ', there are two possibilities. If $h_s > \mathcal{H}_s(\mu')$, in-store customers purchase both fresh and less fresh products and $W_s^C(\mu') = \frac{(\mu')^2}{\Lambda+2\mu'} < \frac{\mu^2}{\Lambda+2\mu}$ (since function $\frac{x^2}{\Lambda+2x}$ is strictly increasing in x for $x > 0$), so waste decreases. If $h_s \leq \mathcal{H}_s(\mu')$, in-store customers only purchase fresh products and $W_s^C(\mu') = \mu'/2$; store waste increases, $W_s^C(\mu') \geq W_s^C(\mu)$, if and only if $\frac{\mu'}{2} \geq \frac{\mu^2}{\Lambda+2\mu}$, i.e., $\mu' \geq \frac{2\mu^2}{\Lambda+2\mu}$.

To obtain the open interval in the statement, it remains to characterize the strict condition $h_s < \mathcal{H}_s(\mu')$ when $h_s > \mathcal{H}_s(\mu)$. By Lemma 1 and the standing positive-profit assumption, the grocer earns positive profit while selling both freshness levels at μ, so $c < p^C \leq V_L$. Hence, for $x \in \{\mu, \mu'\}$,

$$\mathcal{H}_s(x) = \frac{V_F - 2V_L}{2} + \frac{c(\Lambda+x)}{2(\Lambda+2x)} < \frac{V_F - V_L}{2}.$$

Consequently, $h_s < \mathcal{H}_s(\mu')$ is equivalent to

$$h_s < \frac{V_F - 2V_L}{2} + \frac{c(\Lambda+\mu')}{2(\Lambda+2\mu')}.$$

Moreover, $h_s > \mathcal{H}_s(\mu)$ implies $2V_F - 4V_L - 4h_s + c < -c\Lambda/(\Lambda+2\mu) \leq 0$. Solving the preceding inequality therefore gives

$$\mu' < \frac{\Lambda(2h_s - V_F + 2V_L - c)}{2V_F - 4V_L - 4h_s + c}.$$

Combining this condition with the waste comparison above, for $h_s > \mathcal{H}_s(\mu)$ set

$$\mathcal{M}_{c1} = \frac{2\mu^2}{\Lambda+2\mu}, \qquad \mathcal{M}_{c2} = \frac{\Lambda(2h_s - V_F + 2V_L - c)}{2V_F - 4V_L - 4h_s + c}.$$

Store waste increases when $\mu' \in (\mathcal{M}_{c1}, \mathcal{M}_{c2})$. This interval is empty if $\mathcal{M}_{c2} \leq \mathcal{M}_{c1}$.

Omnichannel model. When the conventional optimum is fresh-only, recall from Lemma 2 and its proof that, at spoilage rate μ, customers use BOPS when $h_o < \mathcal{H}_o(\mu)$ and do not use BOPS when $h_o \geq \mathcal{H}_o(\mu)$, where $\mathcal{H}_o(\mu) = \min\{h_{o1}(\mu), h_{o2}(\mu)\}$ with

$$h_{o1}(\mu) = \frac{(\Lambda+\mu)V_F + \mu V_L}{\Lambda+2\mu} - \alpha h_s - \frac{V_F - 2h_s + c}{2} + \frac{c\mu}{2(\Lambda+2\mu)},$$
$$h_{o2}(\mu) = \frac{\mu}{\Lambda+2\mu}\left(V_L - \frac{V_F - 2h_s + c}{2} + \frac{c\mu}{2(\Lambda+2\mu)}\right) + (1-\alpha)h_s.$$

When customers use BOPS (i.e., $\lambda_F = \lambda_L = \Lambda$), the store waste is $W_s^B(\mu) = \mu\theta_L = \frac{\mu^2}{\Lambda+2\mu}$; when no customer uses BOPS, they purchase only fresh products in store (i.e., $\lambda_F = \Lambda$ and $\lambda_L = 0$) and the store waste is $W_s^B(\mu) = \frac{1}{2}\mu$. Since $\frac{x^2}{\Lambda+2x} < \frac{x}{2}$ for $x > 0$ and both are strictly increasing in x, we have $\frac{(\mu')^2}{\Lambda+2\mu'} < \frac{\mu^2}{\Lambda+2\mu}$ and $\frac{(\mu')^2}{\Lambda+2\mu'} < \frac{\mu'}{2} < \frac{\mu}{2}$ for $x > 0$. We discuss the two cases $h_s < \mathcal{H}_s(\mu)$ and $h_s \geq \mathcal{H}_s(\mu)$ separately.

- $h_s < \mathcal{H}_s(\mu)$: Since $h_s < \mathcal{H}_s(\mu) \leq \mathcal{H}_s(\mu')$, the conventional model sets price so that customers purchase only fresh products at both μ and μ'. Depending on whether customers use BOPS at μ or μ', the increase in store waste can only happen when customers use BOPS at μ and do not use BOPS at μ': the other three combinations leave $W_s^B(\mu) > W_s^B(\mu')$. Under which, $W_s^B(\mu) = \frac{\mu^2}{\Lambda+2\mu}$ and $W_s^B(\mu') = \frac{1}{2}\mu'$. So, $W_s^B(\mu') \geq W_s^B(\mu)$ if and only if $\mu' \geq \frac{2\mu^2}{\Lambda+2\mu}$. In addition, this only happens when BOPS is used at μ (i.e., $h_o < \mathcal{H}_o(\mu)$) and is not used at μ' (i.e., $h_o \geq \mathcal{H}_o(\mu')$). To eliminate the scenario where $\mathcal{H}_o(\mu') > \mathcal{H}_o(\mu)$, the interval of h_o is set as $\min\{\mathcal{H}_o(\mu), \mathcal{H}_o(\mu')\} < h_o < \mathcal{H}_o(\mu)$. Thus, we set $\mathcal{H}_{o1} = \min\{\mathcal{H}_o(\mu), \mathcal{H}_o(\mu')\}, \mathcal{H}_{o2} = \mathcal{H}_o(\mu)$, and $\mathcal{M}_b = \frac{2\mu^2}{\Lambda+2\mu}$.

  Under this condition, customers use BOPS at μ and do not use BOPS at μ'. As $\mu > \mu'$, we have $h_s < \mathcal{H}_s(\mu) \leq \mathcal{H}_s(\mu')$, which means that the conventional model has customers purchase fresh products only at both μ and μ'. By Lemma 2, we have $\pi_\mu^B > \pi_\mu^C$ (as customers use BOPS at μ) and $\pi_{\mu'}^B = \pi_{\mu'}^C$ (as customers do not use BOPS at μ'). Therefore, $\Delta_{\pi^B} = \pi_{\mu'}^B - \pi_\mu^B < \pi_{\mu'}^C - \pi_\mu^C = \Delta_{\pi^C}$.
- $h_s \geq \mathcal{H}_s(\mu)$: We discuss the store waste and the profit improvement separately.

  Store waste: At μ, selling both freshness levels is an optimal conventional strategy, possibly tied with fresh-only sales when $h_s = \mathcal{H}_s(\mu)$. As shown in the proof of Lemma 2, BOPS yields strictly higher profit than this full-sale strategy under $h_b < h_s$. Hence, customers use BOPS at μ, giving $W_s^B(\mu) = \frac{\mu^2}{\Lambda+2\mu}$.

  At μ', there are two possibilities. If $h_s \geq \mathcal{H}_s(\mu')$, a conventional full-sale strategy is optimal, possibly tied with fresh-only sales at equality. By the same dominance argument, customers use BOPS and $W_s^B(\mu') = \frac{(\mu')^2}{\Lambda+2\mu'} < W_s^B(\mu)$. If $h_s < \mathcal{H}_s(\mu')$, the conventional optimum is fresh-only,

and customers still use BOPS whenever $h_o < \mathcal{H}_o(\mu')$, again giving $W_s^B(\mu') = \frac{(\mu')^2}{\Lambda+2\mu'} < W_s^B(\mu)$. Thus, for the fixed $\mu' < \mu$, store waste decreases whenever $h_o < \mathcal{H}_o(\mu')$.

Profit improvement: Fix $\mu' \in (\mathcal{M}_{c1}, \mathcal{M}_{c2})$. We calculate the profit improvement Δ_{π^C} and Δ_{π^B}.

— *Conventional model*: The conventional grocer sets its optimal price so that customers purchase both fresh and less fresh products at μ and purchase only fresh products at μ'. Thus, by Lemma 1,

$$\begin{aligned}\Delta_{\pi^C} = \pi^C(\mu') - \pi^C(\mu) &= \left[\frac{\Lambda}{2}(V_F - 2h_s - c) - c\frac{\mu'}{2}\right] - \left[\Lambda\left(p_\ell^C(\mu) - c\right) - c\frac{\mu^2}{\Lambda+2\mu}\right] \\ &= \left[\Lambda\left(\frac{V_F - 2h_s}{2} + \frac{c}{2}\frac{\Lambda+\mu'}{\Lambda+2\mu'} - c\right) - c\frac{(\mu')^2}{\Lambda+2\mu'}\right] - \left[\Lambda\left(p_\ell^C(\mu) - c\right) - c\frac{\mu^2}{\Lambda+2\mu}\right], \\ &= \left[\Lambda\left(\gamma(\mu') - c\right) - c\frac{(\mu')^2}{\Lambda+2\mu'}\right] - \left[\Lambda\left(p_\ell^C(\mu) - c\right) - c\frac{\mu^2}{\Lambda+2\mu}\right]\end{aligned}$$

where $p_\ell^C(\mu) = \min\{V_L, \frac{(\Lambda+\mu)V_F + \mu V_L}{\Lambda+2\mu} - h_s\}$ and, for $x > 0$, we let $\gamma(x) = \frac{V_F - 2h_s}{2} + \frac{c}{2}\frac{\Lambda+x}{\Lambda+2x}$.

— *Omnichannel model*: By Lemma 2, when $h_o < \mathcal{H}_o(\mu')$ (i.e., customers use BOPS at both μ and μ'),

$$\Delta_{\pi^B} = \pi^B(\mu') - \pi^B(\mu) = \left[\Lambda(p^B(\mu') - c) - \frac{c(\mu')^2}{\Lambda+2\mu'}\right] - \left[\Lambda(p^B(\mu) - c) - \frac{c\mu^2}{\Lambda+2\mu}\right].$$

Recall from the proof of Lemma 2, we have $p^B(\mu) = \min\{p_1(\mu), p_2(\mu)\}$, where

$$p_1(\mu) = \frac{(\Lambda+\mu)V_F + \mu V_L}{\Lambda+2\mu} - h_b, \quad p_2(\mu) = V_L + \frac{\Lambda+2\mu}{\mu}(h_s - h_b).$$

Note that, for a fixed μ, both $p_1(\mu)$ and $p_2(\mu)$ are strictly decreasing in h_o and $p_2(\mu) \le p_1(\mu)$ if and only if

$$h_o \ge h_{o3}(\mu) \triangleq h_s\left(\frac{\Lambda+2\mu}{\Lambda+\mu} - \alpha\right) - \frac{\mu(V_F - V_L)}{\Lambda+2\mu}.$$

We see that p_2 decreases faster in h_o than p_1, which means that $p_2(\mu) \le p_1(\mu)$ (i.e., $p^B(\mu) = p_2(\mu)$) for sufficiently large h_o. If, in addition, $h_o < \min\{\mathcal{H}_o(\mu'), h_{o3}(\mu')\}$, then customers use BOPS at μ' and $p^B(\mu') = p_1(\mu')$. For $h_o \ge h_{o3}(\mu)$, we have $p^B(\mu) = p_2(\mu)$.

We now compare the profit improvements under $h_{o3}(\mu) \le h_o < \min\{\mathcal{H}_o(\mu'), h_{o3}(\mu')\}$. We have

$$\begin{aligned}\Delta_{\pi^B} - \Delta_{\pi^C} &= \Lambda\left[p_1(\mu') - p_2(\mu) - \gamma(\mu') + p_\ell^C(\mu)\right] \\ &> \Lambda\left[p_1(\mu') - p_2(\mu) - \gamma(\mu') + \gamma(\mu)\right].\end{aligned}$$

The strict inequality follows because the nonempty switching interval has $h_s > \mathcal{H}_s(\mu)$ and $p_\ell^C(\mu) = V_L$, so $p_\ell^C(\mu) - \gamma(\mu) = h_s - \mathcal{H}_s(\mu) > 0$. Thus, a sufficient condition for $\Delta_{\pi^B} > \Delta_{\pi^C}$ is

$G(h_o) \triangleq p_1(\mu') - p_2(\mu) - \gamma(\mu') + \gamma(\mu) \geq 0$. Substituting the expressions for $p_1(\mu')$, $p_2(\mu)$, $\gamma(\mu')$, and $\gamma(\mu)$ gives

$$G(h_o) = (V_F - V_L)\frac{\Lambda + \mu'}{\Lambda + 2\mu'} - \left[\alpha + \frac{\Lambda + 2\mu}{\mu}(1-\alpha)\right] h_s - \frac{c}{2}\left(\frac{\mu}{\Lambda + 2\mu} - \frac{\mu'}{\Lambda + 2\mu'}\right) + \frac{\Lambda + \mu}{\mu} h_o.$$

Note that $G(h_{o3}(\mu)) = [V_F - V_L - \frac{c}{2}](\frac{\mu}{\Lambda+2\mu} - \frac{\mu'}{\Lambda+2\mu'})$. Therefore, given that $G(h_o)$ is linear in h_o with a constant slope $\frac{\Lambda+\mu}{\mu}$, $G(h_o)$ can be re-expressed with $h_{o3}(\mu)$ as

$$G(h_o) = \left(V_F - V_L - \frac{c}{2}\right)\left(\frac{\mu}{\Lambda + 2\mu} - \frac{\mu'}{\Lambda + 2\mu'}\right) + \frac{\Lambda + \mu}{\mu}(h_o - h_{o3}(\mu)),$$

which follows that $G(h_o) \geq 0$ holds when $h_o \geq \mathcal{H}_{o4}$, where

$$\mathcal{H}_{o4} \triangleq h_{o3}(\mu) + \left[\frac{c}{2} - (V_F - V_L)\right]^+ \left(\frac{\mu}{\Lambda + 2\mu} - \frac{\mu'}{\Lambda + 2\mu'}\right)\frac{\mu}{\Lambda + \mu}.$$

For the fixed μ', set $\mathcal{H}_{o3} = \min\{\mathcal{H}_o(\mu'), h_{o3}(\mu')\}$ and let $\mathcal{H}_{o4}$ be defined above. Then $h_o < \mathcal{H}_{o3}$ ensures the store-waste reduction, and the additional conditions $\mu' \in (\mathcal{M}_{c1}, \mathcal{M}_{c2})$ and $h_o \geq \mathcal{H}_{o4}$ ensure $\Delta_{\pi^B} > \Delta_{\pi^C}$. This proves part (ii). Adding the discussion of the conventional model and the omnichannel model together completes the proof. □

Proof of Proposition 3. The argument applies identically to the conventional and omnichannel models, so we drop the superscripts $\cdot^C$ and $\cdot^B$ except where the two differ.

Recall the taxed profit $\pi_T(p)$ from Equation (12). For a fixed price p, the tax enters $\pi_T(p)$ only through the term $-(c+T)W_{s,T}$, giving

$$\frac{\partial \pi_T(p)}{\partial T} = -W_{s,T} \leq 0. \tag{A.1}$$

Thus the taxed profit is weakly decreasing in T at every price, and so is the grocer's optimal profit.

We first show that a higher tax can only shift the grocer from the fresh-only regime to the full-sale regime, never the reverse. By Lemmas 1 and 2, for either the conventional or omnichannel model, the grocer's operating equilibrium falls in one of two sales regimes. For fresh-only regime, we have $\lambda_F = \Lambda$ and $\lambda_L = 0$, giving $W_{s,T} = \mu\theta_L = \frac{\mu}{2}$. For full-sale regime, we have $\lambda_F = \lambda_L = \Lambda$, giving $W_{s,T} = \mu\theta_L = \frac{\mu^2}{\Lambda+2\mu}$. Within each sales regime, the feasible prices do not depend on T and the store-waste rate is constant across prices. Thus, maximizing profit within a regime does not change its slope in T. Since $\frac{\mu^2}{\Lambda+2\mu} < \frac{\mu}{2}$, Equation (A.1) implies that the optimal profit within the fresh-only regime decreases faster in T than that within the full-sale regime. Hence, as T rises, the grocer can only switch from fresh-only sales to full sales, never the reverse. If only one sales regime is feasible, no regime switch occurs.

We then discuss how imposing a tax affects the store waste and the customer waste separately.

Effect on store waste. If the tax leaves the sales regime unchanged, then $W_{s,T} = W_{s,0}$. If it shifts the grocer from the fresh-only regime to the full-sale regime, store waste decreases from $\frac{\mu}{2}$ to $\frac{\mu^2}{\Lambda+2\mu}$. In either case, $W_{s,T} \leq W_{s,0}$.

Effect on customer waste. Recall the customer-waste rate from Equation (13), we evaluate it in the two regimes. For the fresh-only regime, we have $\lambda_F = \Lambda$ and $\lambda_L = 0$. Then, we denote W_c^f as the customer waste in this regime and $W_c^f = \frac{\Lambda}{2}(\frac{\mu_c}{\rho+\mu_c})^2$. For the full-sale regime, we have $\lambda_F = \lambda_L = \Lambda$. Then, we denote W_c^ℓ as the customer waste in this regime and $W_c^\ell = \Lambda(\frac{\mu_c}{\rho+\mu_c})^2(\frac{\Lambda+\mu}{\Lambda+2\mu}) + \Lambda(\frac{\mu_c}{\rho+\mu_c})(\frac{\mu}{\Lambda+2\mu})$. Taking the difference between the customer waste in these two regimes, we have

$$W_c^\ell - W_c^f = \Lambda \frac{\mu_c}{\rho+\mu_c}\left[\frac{\mu_c}{\rho+\mu_c}\frac{\Lambda+\mu}{\Lambda+2\mu} + \frac{\mu}{\Lambda+2\mu} - \frac{1}{2}\frac{\mu_c}{\rho+\mu_c}\right] = \frac{\Lambda}{\Lambda+2\mu}\left[\frac{\Lambda}{2}\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \frac{\mu\mu_c}{\rho+\mu_c}\right] > 0,$$

using $\frac{\Lambda+\mu}{\Lambda+2\mu} - \frac{1}{2} = \frac{\Lambda}{2(\Lambda+2\mu)}$. Thus, the full-sale regime has strictly higher customer waste than the fresh-only regime. Since the tax can only switch the grocer from fresh-only sales to full sales, or leave the regime unchanged, it follows that $W_{c,T} \geq W_{c,0}$.

Since the above discussion does not rely on whether the model is the conventional model or the omnichannel model, the conclusions hold for both models. That is, $W_{s,T}^C \leq W_{s,0}^C$, $W_{s,T}^B \leq W_{s,0}^B$, $W_{c,T}^C \geq W_{c,0}^C$, and $W_{c,T}^B \geq W_{c,0}^B$. □

Proof of Proposition 4. To prove the statement, we first derive the threshold for the conventional model where $\Delta_{W^C} \geq 0$ if and only if $\mu_c \geq \bar{\mu}_c^C$. We then derive the similar threshold $\bar{\mu}_C^B$ for the omnichannel model. Finally, we pick $\bar{\mu}_C = \max\{\bar{\mu}_c^C, \bar{\mu}_c^B\}$ to complete the proof.

Conventional model. Recall that in the fresh-only regime, customers purchase only fresh products and $(\lambda_F, \lambda_L) = (\Lambda, 0)$, $\theta_F = \theta_L = \frac{1}{2}$. Using the customer-waste expression in Equation 13, the total waste in the fresh only regime is $W_t^f = \frac{\mu}{2} + \frac{\Lambda}{2}\left(\frac{\mu_c}{\rho+\mu_c}\right)^2$. In this regime, the optimal price is $V_F - 2h_s$ with $\pi^f(T) = \frac{\Lambda}{2}(V_F - 2h_s - c) - \frac{(c+T)\mu}{2}$. In the full-sale regime, customers purchase both fresh and less-fresh products, so $(\lambda_F, \lambda_L) = (\Lambda, \Lambda)$ with $\theta_F = \frac{\Lambda+\mu}{\Lambda+2\mu}, \theta_L = \frac{\mu}{\Lambda+2\mu}$. Total waste is therefore $W_t^\ell = \frac{\mu^2}{\Lambda+2\mu} + \frac{\Lambda(\Lambda+\mu)}{\Lambda+2\mu}\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \frac{\Lambda\mu}{\Lambda+2\mu}\frac{\mu_c}{\rho+\mu_c}$. In this regime, the optimal price is $p_\ell = \min\left\{\frac{(\Lambda+\mu)V_F + \mu V_L}{\Lambda+2\mu} - h_s,\, V_L\right\}$ and $\pi^\ell(T) = \Lambda(p_\ell - c) - \frac{(c+T)\mu^2}{\Lambda+2\mu}$.

Since the tax does not enter customer utilities, then it does not change the feasible price set in either regime. Moreover, within each regime, the store-waste rate is independent of price; the tax therefore subtracts the same amount from profit at every feasible price. Consequently, the optimal prices in each regime are independent of T.

Define the threshold of tax T^C as follows:

$$T^C = \frac{2(\Lambda+2\mu)}{\Lambda\mu}\left[\pi^f(0) - \pi^\ell(0)\right] = \frac{(\Lambda+2\mu)(V_F - 2h_s - 2p_\ell) + c(\Lambda+\mu)}{\mu}.$$

Then, define the threshold $\bar{\mu}_c^C$ as

$$\bar{\mu}_c^C = \begin{cases} \rho\sqrt{\dfrac{\mu}{\Lambda+\mu}}, & 0 \le T^C < T, \\ 0, & \text{otherwise.} \end{cases}$$

To show $\Delta W^C \ge 0$, we discuss case by case. When the fresh only regime is the equilibrium in the conventional model, we have $\pi^f(0) \ge \pi^\ell(0)$ when there is no tax, which implies that $T^C \ge 0$. For $0 \le T < T^C$, we have $\bar{\mu}_c^C = 0$ and

$$\pi^\ell(T) - \pi^f(T) = \pi^\ell(0) - \pi^f(0) + T\left(\frac{\mu}{2} - \frac{\mu^2}{\Lambda+2\mu}\right) = \pi^\ell(0) - \pi^f(0) + \frac{\Lambda\mu}{2(\Lambda+2\mu)}T \le 0,$$

which implies that the fresh-only regime stays to be optimal when inducing a tax $0 \le T < T^C$ and $\Delta_{W^C} = 0$ if and only if $\mu_c \ge 0 = \bar{\mu}_c^C$. At $T = T^C$, the grocer is indifferent and selects the fresh-only regime by the stated convention. When $T > T^C \ge 0$, the threshold $\bar{\mu}_c^C = \rho\sqrt{\frac{\mu}{\Lambda+\mu}}$. The full-sale regime becomes optimal and the change of the total waste

$$\Delta_{W^C} = W_t^\ell - W_t^f = \frac{\Lambda}{2(\Lambda+2\mu)}\left[\Lambda\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + 2\mu\frac{\mu_c}{\rho+\mu_c} - \mu\right] = \frac{\Lambda\left[(\Lambda+\mu)\mu_c^2 - \mu\rho^2\right]}{2(\Lambda+2\mu)(\rho+\mu_c)^2} \ge 0$$
$$\iff \mu_c \ge \rho\sqrt{\frac{\mu}{\Lambda+\mu}} = \bar{\mu}_c^C \text{ when } T > T^C \ge 0.$$

When the full sale regime is the equilibrium in the conventional model, we have $\pi^\ell(0) > \pi^f(0)$, which means that $T^C < 0$ and $\bar{\mu}_c^C = 0$. Then, for every $T > 0$,

$$\pi^\ell(T) - \pi^f(T) = \pi^\ell(0) - \pi^f(0) + \frac{\Lambda\mu}{2(\Lambda+2\mu)}T > 0.$$

Thus, the full sale regime stays to be optimal regardless of the amount of imposed tax, which implies $\Delta_{W^C} = 0$ if and only if $\mu_c \ge 0 = \bar{\mu}_c^C$. Combining two cases above, we conclude that $\Delta_{W^C} \ge 0$ if and only if $\mu_c > \bar{\mu}_c^C$.

Omnichannel model. By Lemma 2, the two feasible regimes are fresh-only in-store purchases and full-sale BOPS purchases, which have the same stationary distribution as their correspondences in the conventional model. Thus, the store waste, and customer waste are exactly those calculated in the conventional model, implying that their total-waste rates are W_t^f and W_t^ℓ, respectively. Since imposing tax does not affect the optimal prices in each feasible regime, then the optimal price in the fresh-only in-store regime is $V_F - 2h_s$ with $\pi^f(T) = \frac{\Lambda}{2}(V_F - 2h_s - c) - \frac{(c+T)\mu}{2}$ and the optimal price in the full-sale BOPS regime is $p_b = \min\left\{\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - h_b,\, V_L + \frac{\Lambda+2\mu}{\mu}(h_s - h_b)\right\}$. with $\pi^b(T) = \Lambda(p_b - c) - \frac{(c+T)\mu^2}{\Lambda+2\mu}$. Define the threshold of tax T^B as follows:

$$T^B = \frac{2(\Lambda+2\mu)}{\Lambda\mu}\left[\pi^f(0) - \pi^b(0)\right] = \frac{(\Lambda+2\mu)(V_F - 2h_s - 2p_b) + c(\Lambda+\mu)}{\mu} \ge 0.$$

Then, define the threshold $\bar{\mu}_c^B$ as

$$\bar{\mu}_c^B = \begin{cases} \rho\sqrt{\dfrac{\mu}{\Lambda+\mu}}, & 0 \le T^B < T, \\ 0, & \text{otherwise.} \end{cases}$$

To show $\Delta W^C \ge 0$, we discuss case by case. When the fresh only regime is the equilibrium in the omnichannel model, we have $\pi^f(0) \ge \pi^b(0)$, which means $T^B \ge 0$. For $0 \le T \le T^B$, we have

$$\pi^b(T) - \pi^f(T) = \pi^b(0) - \pi^f(0) + T\left(\frac{\mu}{2} - \frac{\mu^2}{\Lambda+2\mu}\right) = \pi^b(0) - \pi^f(0) + \frac{\Lambda\mu}{2(\Lambda+2\mu)}T \le 0,$$

and $\bar{\mu}_c^B = 0$. Then, the fresh-only regime stays to be optimal when inducing a tax $0 \le T \le T^B$ and $\Delta_{W^B} = 0$ if and only if $\mu_c \ge 0 = \bar{\mu}_c^B$. When $T > T^B$, the full-sale regime becomes optimal

$$\Delta_{W^B} = W_t^\ell - W_t^f \ge 0 \iff \mu_c \ge \rho\sqrt{\frac{\mu}{\Lambda+\mu}} = \bar{\mu}_c^B \text{ when } T > T^B \ge 0.$$

When the full sale regime is the equilibrium in the omnichannel model, we have $\pi^b(0) > \pi^f(0)$, which implies that $T^B < 0$ and $\bar{\mu}_c^B = 0$. Then, for every $T > 0$, $\pi^b(T) - \pi^f(T) = \pi^b(0) - \pi^f(0) + \frac{\Lambda\mu}{2(\Lambda+2\mu)}T > 0$. Thus, the full sale regime stays to be optimal regardless of the amount of imposed tax, which implies $\Delta_{W^B} = 0$ if and only if $\mu_c \ge 0 = \bar{\mu}_c^B$. Combining thresholds for both models, we have $\bar{\mu}_c = \max\{\mu_c^C, \mu_c^B\}$, and $\Delta_{W^B} \ge 0$ if and only if $\mu_c > \bar{\mu}_c$.

We show the rest of the statement in Proposition 4 under $\mu_c \ge \bar{\mu}_c$. Notice $h_o < (1-\alpha)h_s$ implies

$$\frac{(\Lambda+\mu)V_F + \mu V_L}{\Lambda+2\mu} - h_b > \frac{(\Lambda+\mu)V_F + \mu V_L}{\Lambda+2\mu} - h_s \text{ and } V_L + \frac{\Lambda+2\mu}{\mu}(h_s - h_b) > V_L.$$

Thus, $p_b \ge p_\ell$ and $\pi^b(T) - \pi^\ell(T) = \Lambda(p_b - p_\ell) > 0$ for every $T \ge 0$.

When $h_o \le \mathcal{H}_o$, by Lemma 2, the omnichannel grocer already serves both freshness levels through BOPS at $T = 0$. Thus, $\Delta_{W^B} = 0$ for every $T > 0$, and $0 = \Delta_{W^B} \le \Delta_{W^C}$ since $\Delta_{W^C} \ge 0$ when $\mu_c \ge \bar{\mu}_c$.

When $h_o > \mathcal{H}_o$, the omnichannel no-tax equilibrium is fresh-only, which implies $\pi^f(0) \ge \pi^b(0)$. If the conventional no-tax equilibrium is full sale, then we have $\pi^\ell(0) \ge \pi^f(0)$. Combining with $\pi^b(0) - \pi^\ell(0) > 0$, we have $\pi^b(0) > \pi^f(0)$, reaching a contradiction. Thus, the conventional no-tax equilibrium must also be fresh only. Since $p_b \ge p_\ell$, we have $T^C - T^B = \frac{2(\Lambda+2\mu)}{\Lambda\mu}[\pi^b(0) - \pi^\ell(0)] = \frac{2(\Lambda+2\mu)}{\mu}(p_b - p_\ell) > 0$, implying $0 \le T^B < T^C$. Thus, for $0 \le T \le T^B$, both grocers remain fresh-only and $\Delta_{W^B} = \Delta_{W^C} = 0$. For $T^B < T \le T^C$, the omnichannel grocer has switched to full sale, while the conventional grocer remains fresh-only. Then, $\Delta_{W^B} \ge \Delta_{W^C} = 0$ since $\Delta_{W^B} \ge 0$ when $\mu_c \ge \bar{\mu}_c$. For $T > T^C$, both grocers have switched and $\Delta_{W^B} = W_t^\ell - W_t^f = \Delta_{W^C} \ge 0$. Therefore, we conclude that $\Delta_{W^B} \ge \Delta_{W^C} \ge 0$ when $h_o > \mathcal{H}_o$ and $\mu_c \ge \bar{\mu}_c$. □

A.2. Model Extension of Shelf Size

In this section, we show that the key insights from our main model (Propositions 2-4) also hold when there are at most two products on the shelf.

A.2.1. Equilibrium Outcomes under Different Shelf Size

To show the main insights hold in the two-product extension with a general picking probability, we first prove the equilibirum results similar to Lemmas 1 and 2 in the main model.

LEMMA A2.1. *In the conventional model, the grocer's equilibrium profit is positive if and only if* $h_s \le \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu}$ *and* $c < \max\left\{\frac{3\Lambda(V_F-\frac{4}{3}h_s)}{3\Lambda+4\mu}, \frac{\Lambda}{\Lambda+\frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}}\min\left\{V_L, \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu}-h_s\right\}\right\}$. *Under this condition, the equilibrium product price* p^C *and the associated profit* π^C *are characterized as follows.*

(i) If $0 < h_s \le \min\left\{\frac{3}{4}(V_F-V_L), \frac{3V_F-4V_L}{4}+\frac{c(3\Lambda+4\mu)}{4(3\Lambda+8\mu)}\right\}$, *then* $p^C = V_F - \frac{4}{3}h_s$ *and* $\pi^C = \frac{3\Lambda}{4}(V_F-\frac{4}{3}h_s - c) - c\mu$.

(ii) If $\min\left\{\frac{3}{4}(V_F-V_L), \frac{3V_F-4V_L}{4}+\frac{c(3\Lambda+4\mu)}{4(3\Lambda+8\mu)}\right\} < h_s \le \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu}$, *then* $p^C = \min\left\{V_L, \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu}-h_s\right\}$ *and* $\pi^C = \Lambda(p^C-c) - c\frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}$.

For the rest of the analysis in this section, we assume the conditions in Lemma A2.1 (i.e., $h_s \le \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu}$ and $c < \max\left\{\frac{3\Lambda(V_F-\frac{4}{3}h_s)}{3\Lambda+4\mu}, \frac{\Lambda}{\Lambda+\frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}}\min\left\{V_L, \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu}-h_s\right\}\right\}$) to ensure positive profit for the grocer in this benchmark.

LEMMA A2.2. *In the omnichannel model with two products, there exists a threshold* $\mathcal{H}_o^m$, *such that*

1. *If* $h_o < \mathcal{H}_o^m$, *then it is strictly profitable for the grocer to offer BOPS (i.e.,* $\pi^B > \pi^C$*). Under this condition, the equilibrium price is*

$$p^B = \min\left\{V_F - \frac{2\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F-V_L) - h_b,\right.$$
$$\left. V_L - \frac{(1-\psi)(\Lambda+2\mu)}{\mu}(V_F-V_L) + \frac{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}{2\mu^2}(h_s-h_b)\right\},$$

   and the associated profit is

$$\pi^B = \Lambda(p^B-c) - \frac{2c\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}.$$

2. *If* $h_o > \mathcal{H}_o^m$, *then the grocer does not offer BOPS, and consequently the equilibrium is identical to that in the conventional model.*

A.2.2. Impact of Reducing the Store Spoilage Rate

We show that the main insights in Proposition 2 can be extended to this two product model. Note the store-waste rate is $W_s(\mu) = \mu\theta_{(F,L)} + 2\mu\theta_{(L,L)}$. Similar to the main model, we can define π_μ^C, Δ_{π^C} and π_μ^B, Δ_{π^B}.

PROPOSITION A2.1. *There exists a threshold* $\mathcal{H}_s^m$, *such that*

(i) If $h_s < \mathcal{H}_s^m$, then

- *In the conventional model, store waste always decreases when μ decreases to μ', i.e., $W_s^C(\mu) \geq W_s^C(\mu')$.*
- *With BOPS, store waste increases (i.e., $W_s^B(\mu) \leq W_s^B(\mu')$) if $h_o \in (\mathcal{H}_{o1}^m, \mathcal{H}_{o2}^m)$ and $\mu' \in (\mathcal{M}_b^m, \mu)$ for some thresholds $\mathcal{H}_{o1}^m, \mathcal{H}_{o2}^m$, and $\mathcal{M}_b^m$; under this condition, $\Delta_{\pi^B} < \Delta_{\pi^C}$.*

(ii) If $h_s \geq \mathcal{H}_s^m$, then

- *In the conventional model, store waste increases (i.e., $W_s^C(\mu) \leq W_s^C(\mu')$) if $\mu' \in (\mathcal{M}_{c1}^m, \mathcal{M}_{c2}^m)$ for some thresholds $\mathcal{M}_{c1}^m$ and $\mathcal{M}_{c2}^m$.*
- *With BOPS, if $h_o < \mathcal{H}_{o3}^m$ for some threshold $\mathcal{H}_{o3}^m$, then*
  - *store waste decreases (i.e., $W_s^B(\mu) \geq W_s^B(\mu')$) for any $\mu' < \mu$;*
  - *$\Delta_{\pi^B} > \Delta_{\pi^C}$ if $\mu' \in (\mathcal{M}_{c1}^m, \mathcal{M}_{c2}^m)$ and $h_o \geq \mathcal{H}_{o4}^m$ for some threshold $\mathcal{H}_{o4}^m$.*

Proposition A2.1 confirms that the main insight of Proposition 2 persists in the two-unit shelf model: when store-shopping hassle is small, reducing the store spoilage rate always lowers store waste in the conventional model but can increase it in the omnichannel model, in which case the omnichannel profit gain is smaller; when store-shopping hassle is large, the same improvement can increase store waste in the conventional model while reducing it in the omnichannel model, in which case the omnichannel profit gain can be larger.

A.2.3. Impact of Retail Waste Tax

Similarly, as the main model, we impose a retail waste tax T per unit that spoils in store. For a feasible stationary outcome, the store-waste rate is $W_{s,T} = \mu\theta_{(F,L)} + 2\mu\theta_{(L,L)}$, and the grocer's profit is

$$\pi_T(p) = (p-c)\Big[(\lambda_{F,s}+\lambda_o)(\theta_{(F,F)}+\theta_{(F,L)}) + (\lambda_{L,s}+\lambda_o)\theta_{(L,L)}\Big] - (c+T)W_{s,T}.$$

As in the main model, a unit purchased fresh or less fresh spoils at home before consumption with probability $[\mu_c/(\rho+\mu_c)]^2$ or $\mu_c/(\rho+\mu_c)$, respectively. Customer waste and total waste are therefore

$$\begin{aligned} W_{c,T} &= \left(\frac{\mu_c}{\rho+\mu_c}\right)^2 \Big[\lambda_{F,s}(\theta_{(F,F)}+\theta_{(F,L)}) + \lambda_o(\theta_{(F,F)}+\psi\theta_{(F,L)})\Big] \\ &\quad + \frac{\mu_c}{\rho+\mu_c}\Big[\lambda_{L,s}\theta_{(L,L)} + \lambda_o\big(\theta_{(L,L)} + (1-\psi)\theta_{(F,L)}\big)\Big], \\ W_{t,T} &= W_{s,T} + W_{c,T}. \end{aligned}$$

Let π_T^C and π_T^B denote the grocer's optimal profit under the conventional and BOPS models, respectively, when the unit tax is T. For $j \in \{C, B\}$, let $W_{s,T}^j$, $W_{c,T}^j$, and $W_{t,T}^j$ denote the corresponding waste rates at the equilibrium in each model, and define $\Delta_{W^j} = W_{t,T}^j - W_{t,0}^j$.

PROPOSITION A2.2. *In both the conventional and omnichannel models, imposing tax weakly decreases store waste (i.e., $W^C_{s,T} \le W^C_{s,0}$ and $W^B_{s,T} \le W^B_{s,0}$) and weakly increases customer waste (i.e., $W^C_{c,T} \ge W^C_{c,0}$ and $W^B_{c,T} \ge W^B_{c,0}$).*

Proposition A2.2 is consistent with the result in Proposition 3. Let $\mathcal{H}^m_o$ be the threshold in the revised Lemma A2.2, evaluated at $T=0$, and let $\Delta_{W^j} = W^j_{t,T} - W^j_{t,0}$ for $j \in \{C, B\}$.

PROPOSITION A2.3. *In both the conventional and omnichannel models, increasing tax weakly increases the total waste if $\mu \ge \overline{\mu}^m_c$. Under this condition*

(i) If $h_o < \mathcal{H}^m_o$, then $\Delta_{W^B} = 0 \le \Delta_{W^C}$.

(ii) If $h_o > \mathcal{H}^m_o$, then $\Delta_{W^B} \ge \Delta_{W^C} \ge 0$ when $0 \le T < \mathcal{T}^m$.

Proposition A2.3 confirms that the main insight of Proposition 4 also persists in the two-unit shelf model: when household spoilage is sufficiently high, a retail waste tax can increase total waste; when online-shopping hassle is low, this increase is no greater in the omnichannel model than in the conventional model, whereas the comparison can reverse when online-shopping hassle is high.

A.3. Model Extension of Non-Immediate Restocking

In this section, we show that the key insights from our main model (Propositions 2-4) also hold when the restocking does not happen simultaneously.

A.3.1. Equilibrium Outcomes under Non-Immediate Restocking

For simplicity, we use the following shorthand throughout this section.

$$\eta \triangleq \frac{\sigma(\Lambda+2\mu)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}, \qquad \eta_0 \triangleq \frac{\sigma}{2\sigma+\Lambda+\mu}.$$

Here $\eta = \theta_F + \theta_L$ evaluated at $\lambda_F = \lambda_L = \Lambda$ is the probability that a product is available to a customer who is willing to buy either freshness level; $\eta_0 = \theta_F$ evaluated at $\lambda_L = 0$ and $\lambda_F = \Lambda$ is the probability that a fresh unit is on the shelf when only fresh units are sold.

LEMMA A3.1. *Let $\hat{h}_s \triangleq \frac{1}{\Lambda}\Big(\eta_0\big[\Lambda(V_F-c)-c\mu\big] - \eta\big[\Lambda(V_L-c) - \frac{c\mu^2}{\Lambda+2\mu}\big]\Big)$. In the conventional model with non-immediate restocking, the grocer's equilibrium profit is positive if and only if $h_s \le \eta\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$ and $c < \max\left\{\frac{\Lambda(V_F-\frac{h_s}{\eta_0})}{\Lambda+\mu}, \frac{\Lambda(\Lambda+2\mu)}{(\Lambda+\mu)^2}\min\left\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - \frac{h_s}{\eta}\right\}\right\}$. Under this condition, the equilibrium product price p^C and the associated profit π^C are characterized as follows.*

(i) If $0 < h_s \le \min\big\{\eta_0(V_F-V_L),\ \hat{h}_s\big\}$, then $p^C = V_F - \dfrac{h_s}{\eta_0}$ and $\pi^C = \eta_0\big[\Lambda\big(V_F - \frac{h_s}{\eta_0} - c\big) - c\mu\big]$;

(ii) If $\min\big\{\eta_0(V_F-V_L),\ \hat{h}_s\big\} < h_s \le \eta\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$, then $p^C = \min\big\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - \frac{h_s}{\eta}\big\}$ and $\pi^C = \eta\big[\Lambda(p^C-c) - \frac{c\mu^2}{\Lambda+2\mu}\big]$.

For the rest of the analysis in this section, we assume the condition in Lemma A3.1 (i.e., $h_s \le \eta\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}$ and $c < \max\left\{\frac{\Lambda(V_F-\frac{h_s}{\eta_0})}{\Lambda+\mu}, \frac{\Lambda(\Lambda+2\mu)}{(\Lambda+\mu)^2}\min\left\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - \frac{h_s}{\eta}\right\}\right\}$) to ensure positive profit for the grocer in this benchmark.

LEMMA A3.2. *In the omnichannel model with non-immediate restocking, there exists a threshold* $\mathcal{H}_o^r \le (1-\alpha)h_s$ *such that*

(i) if $h_o < \mathcal{H}_o^r$*, then it is strictly profitable for the grocer to offer BOPS (i.e.,* $\pi^B > \pi^C$*). Under this condition, the equilibrium price is*

$$p^B = \min\left\{\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - \frac{h_b}{\eta}, V_L + \frac{\Lambda+2\mu}{\mu}\frac{h_s-h_b}{\eta}\right\},$$

and the associated profit is $\pi^B = \eta\big[\Lambda(p^B-c) - c\mu^2/(\Lambda+2\mu)\big]$.

(ii) if $h_o \ge \mathcal{H}_o^r$*, the grocer does not offer BOPS, and consequently the equilibrium is identical to that in the conventional model.*

A.3.2. Impact of Reducing the Store Spoilage Rate

We show that the main insights in Proposition 2 can be extended to this model with non-immediate restocking. Note the store-waste rate is $W_s(\mu) = \mu\theta_L$. Similar to the main model, we can define π_μ^C, Δ_{π^C} and π_μ^B, Δ_{π^B}.

PROPOSITION A3.1. *There exists a threshold* $\mathcal{H}_s^r$

(i) If $0 < h_s \le \mathcal{H}_s^r$*, then*

- *In the conventional model, store waste decreases when* μ *decreases to* μ'*, i.e.,* $W_s^C(\mu) \ge W_s^C(\mu')$.
- *In the omnichannel model, store waste increases, i.e.,* $W_s^B(\mu) \le W_s^B(\mu')$*, if* $h_o \in (\mathcal{H}_{o1}^r, \mathcal{H}_{o2}^r)$ *and* $\mu' \in (\mathcal{M}_b^r, \mu)$ *for the thresholds defined in the proof. Under these conditions,* $\Delta_{\pi^B} < \Delta_{\pi^C}$.

(ii) If $h_s > \mathcal{H}_s^r$*, then*

- *In the conventional model, store waste increases, i.e.,* $W_s^C(\mu) \le W_s^C(\mu')$*, if* $\mu' \in (\mathcal{M}_{c1}^r, \mathcal{M}_{c2}^r)$*, where the thresholds are defined in the proof.*
- *In the omnichannel model, for each fixed* $\mu' < \mu$*, there exists a threshold* $\mathcal{H}_{o3}^r$*, such that* $h_o < \mathcal{H}_{o3}^r$ *implies* $W_s^B(\mu) \ge W_s^B(\mu')$*. Under this condition,* $\Delta_{\pi^B} > \Delta_{\pi^C}$ *if, in addition,* $\mu' \in (\mathcal{M}_{c1}^r, \mathcal{M}_{c2}^r)$ *and* $h_o \ge \mathcal{H}_{o4}^r$*, where* $\mathcal{H}_{o4}^r$ *is defined in the proof.*

Proposition A3.1 confirms that the main insight of Proposition 2 persists in this model extension: when store-shopping hassle is small, reducing the store spoilage rate always lowers store waste in the conventional model but can increase it in the omnichannel model, in which case the omnichannel profit gain is smaller; when store-shopping hassle is large, the same improvement can increase store waste in the conventional model while reducing it in the omnichannel model, in which case the omnichannel profit gain can be larger.

A.3.3. Impact of Retail Waste Tax

Consider a tax T for which the grocer's optimal operating profit remains positive under both channel configurations. For a feasible stationary outcome under non-immediate restocking, the store-waste rate is $W_{s,T} = \mu\theta_L$, and the grocer's profit is $\pi_T(p) = (p-c)(\lambda_F\theta_F + \lambda_L\theta_L) - (c+T)W_{s,T}$. Here, θ_F, θ_L, and $\theta_\emptyset$ are the stationary probabilities of the fresh, less-fresh, and empty shelf states, respectively; the empty state generates neither sales nor store waste. As in the main model, a unit purchased fresh or less fresh spoils at home before consumption with probability $[\mu_c/(\rho+\mu_c)]^2$ or $\mu_c/(\rho+\mu_c)$, respectively. Customer waste and total waste are therefore

$$W_{c,T} = \left(\frac{\mu_c}{\rho+\mu_c}\right)^2 \lambda_F\theta_F + \frac{\mu_c}{\rho+\mu_c}\lambda_L\theta_L \text{ and } W_{t,T} = W_{s,T} + W_{c,T}.$$

Let π_T^C and π_T^B denote the grocer's optimal profit under the conventional and BOPS models, respectively, when the unit tax is T. For $j \in \{C, B\}$, let $W_{s,T}^j$, $W_{c,T}^j$, and $W_{t,T}^j$ denote the corresponding waste rates at the equilibrium in each model.

Proposition A3.2. *In both the conventional and omnichannel models, imposing tax weakly decreases the store waste (i.e., $W_{s,T}^C \le W_{s,0}^C$ and $W_{s,T}^B \le W_{s,0}^B$) and weakly increases the customer waste (i.e., $W_{c,T}^C \ge W_{c,0}^C$ and $W_{c,T}^B \ge W_{c,0}^B$).*

Proposition A3.2 extends Proposition 3 to non-immediate restocking. Although stockouts change the stationary shelf probabilities, the full-sale regime still generates less store waste and more customer waste than the fresh-only regime. A waste tax makes full sales relatively more profitable and can therefore shift waste from the store to households, yielding the same directional conclusions as in the immediate-restocking model.

Let $\mathcal{H}_o^r$ be the threshold in the revised Lemma A3.2, evaluated at $T = 0$, and let $\Delta_{W^j} = W_{t,T}^j - W_{t,0}^j$ for $j \in \{C, B\}$.

Proposition A3.3. *In both the conventional and omnichannel models, imposing tax weakly increases the total waste if $\mu_c \ge \overline{\mu}_c^r$, where $\overline{\mu}_c^r \triangleq \frac{\rho}{2}\left[\frac{\mu}{\sigma} + \sqrt{\left(\frac{\mu}{\sigma}\right)^2 + \frac{4\mu}{\sigma} + \frac{4\mu}{\Lambda+\mu}}\right]$. Under this condition,*

(i) if $h_o \ge \mathcal{H}_o^r$, then $\Delta_{W^B} \ge \Delta_{W^C} \ge 0$;

(ii) If $h_o < \mathcal{H}_o^r$, then $\Delta_{W^B} = 0 \le \Delta_{W^C}$.

Proposition A3.3 confirms that the main insight of Proposition 4 also persists in this model extension: when household spoilage is sufficiently high, a retail waste tax can increase total waste; when online-shopping hassle is low, this increase is no greater in the omnichannel model than in the conventional model, whereas the comparison reverses when online-shopping hassle is high.

A.4. Model Extension of Heterogeneous Consumer Hassle Cost

In this section, we provide additional analysis of the extended model when the customer hassle cost is heterogeneous. To obtain a sharp characterization of the equilibrium, we specialize the general hassle distribution G of Section 6.3 to the uniform distribution on $[\bar{h}_s-\delta,\bar{h}_s+\delta]$, with $\bar{h}_s>0$ the mean hassle cost and $\delta\in(0,\bar{h}_s)$ the degree of heterogeneity; that is, $G(t)=\frac{t-(\bar{h}_s-\delta)}{2\delta}$ for $t\in[\bar{h}_s-\delta,\bar{h}_s+\delta]$, and 0 or 1 outside. The thresholds $A(p)$, $B(p)$, and $C(p)$ are as defined in Equations (26), (29), and (30); the participation fractions $\Phi(p)$, $\Phi_s(p)$, and $\Phi_b(p)$ as in Equations (27), (31), and (32); and the demand rates as in Equations (28) and (33).

A.4.1. Equilibrium Outcomes under Heterogeneous Hassle

Lemmas A4.1 and A4.2 characterize the equilibrium outcomes of the conventional model and the omnichannel models. Throughout, $\Lambda>0$, $\mu>0$, $V_F>V_L\geq 0$, $\bar{h}_s>0$, and $0<\delta<\bar{h}_s$; in the omnichannel model $0<\alpha<1$ and $h_o\geq 0$. For a candidate less-fresh demand rate $x\in[0,\Lambda]$, let $\theta_F(x)=\frac{x+\mu}{x+2\mu}$ and $\theta_L(x)=\frac{\mu}{x+2\mu}$. To make the dependence on the candidate demand rate explicit, write the thresholds from Equations (26), (29), and (30) as $A(p,x),B(p,x),C(p,x)$, where θ_F,θ_L in the calculation of each has been replaced by $\theta_F(x),\theta_L(x)$ respectively. At an equilibrium with $\lambda_L=x$, these are the thresholds $A(p)$, $B(p)$, and $C(p)$ used in Section 6.3.

Lemma A4.1. *Under the uniform hassle distribution, given any $p\in[0,V_F]$, the subgame in the conventional model has at least one equilibrium. All equilibrium demand rates in the subgame are characterized as follows.*

(i) When $0\leq p\leq V_L$, the equilibrium less-fresh demand rates are the fixed points $x\in[0,\Lambda]$ of $x=\Lambda G(A(p,x))$ and $A(p,x)=\frac{(x+\mu)V_F+\mu V_L}{x+2\mu}-p$. For each such fixed point, $\lambda_F=\lambda_L=x$. The endpoint and interior candidates are characterized by: $\lambda_L=0$ is an equilibrium if and only if $A(p,0)\leq\bar{h}_s-\delta$; $\lambda_L=\Lambda$ is an equilibrium if and only if $A(p,\Lambda)\geq\bar{h}_s+\delta$. A rate $\lambda_L\in(0,\Lambda)$ is an equilibrium if and only if it satisfies

$$2\delta\lambda_L^2+\big(4\delta\mu+\Lambda[(\bar{h}_s-\delta+p)-V_F]\big)\lambda_L+\Lambda\mu\big(2(\bar{h}_s-\delta+p)-V_F-V_L\big)=0. \tag{A.2}$$

(ii) When $V_L<p\leq V_F$, the equilibrium demand rates in the subgame are uniquely given by $\lambda_L=0$ and $\lambda_F=\Lambda G\big(\frac{V_F-p}{2}\big)$. In particular, both demand rates are zero at $p=V_F$.

The grocer evaluates the profit in Equation (2) at every feasible price and its associated equilibrium demand rates, and selects a profit-maximizing pair according to the convention below. Whenever such a selected optimum exists, denote its price and profit by p^C and π^C, respectively.

Lemma A4.2. *Under the uniform hassle distribution, for any $p\in[0,V_F]$, the subgame in the omnichannel model has at least one equilibrium. All equilibrium demand rates in the subgame are characterized as follows.*

(i) When $0 \le p \le V_L$, the equilibrium less-fresh purchase rates are the fixed points $x \in [0, \Lambda]$ of $x = \Lambda G(\max\{A(p,x), C(p,x)\})$. For each such fixed point, $\lambda_F = \lambda_L = x$. If $B(p,x) \ge A(p,x)$, the BOPS participation fraction is zero, and the candidate conditions are those in Lemma A4.1(i). No customer uses the BOPS channel in this branch. If $B(p,x) < A(p,x)$, the channel participation fractions are $\Phi_s(p,x) = G(B(p,x)), \Phi_b(p,x) = G(C(p,x)) - G(B(p,x))$, and the fixed-point equation is $x = \Lambda G(C(p,x))$. Within this branch, $\lambda_L = 0$ is an equilibrium if and only if $C(p,0) \le \bar{h}_s - \delta$, and $\lambda_L = \Lambda$ is an equilibrium if and only if $C(p,\Lambda) \ge \bar{h}_s + \delta$. A rate $\lambda_L \in (0,\Lambda)$ is an equilibrium in this branch if and only if it satisfies

$$\begin{aligned} 2\alpha\delta\lambda_L^2 + \big(4\alpha\delta\mu + \Lambda[\alpha(\bar{h}_s - \delta) + h_o + p - V_F]\big)\lambda_L \\ + \Lambda\mu\big(2\alpha(\bar{h}_s - \delta) + 2h_o + 2p - V_F - V_L\big) = 0. \end{aligned} \tag{A.3}$$

Every endpoint and interior candidate in this branch must also satisfy $B(p,x) < A(p,x)$.

(ii) When $V_L < p \le V_F$, in the subgame, the equilibrium less-fresh purchase rates λ_L are the fixed points $x \in [0,\Lambda]$ of $x = \Lambda\big[G(C(p,x)) - G(B(p,x))\big]^+$. For each such fixed point, the fresh purchase rate is $\lambda_F = x + \Lambda G(\min\{A(p,x), B(p,x)\})$. At $p = V_F$, the unique equilibrium has $\lambda_F = \lambda_L = 0$.

The grocer evaluates the profit in Equation (2) at every feasible price and its associated equilibrium demand rates, and selects a profit-maximizing pair according to the convention below. Whenever such a selected optimum exists, denote its price and profit by p^B and π^B, respectively.

A.4.2. Impact of Reducing the Store Spoilage Rate

To demonstrate that Proposition 2 is robust in the heterogeneous hassle cost setting, we conduct a numerical robustness check. Similar to the main model, we can define $W_s(\mu) = \mu\theta_L$, π_μ^C, Δ_{π^C} and π_μ^B, Δ_{π^B}. We start by discussing how we choose the values for all the model parameters based on empirical evidences.

Parameter description. Before presenting the detailed discussion of the parameters, we note the two normalization:

- Normalizing $v_F = 1$: Given that the model is scale-free in its monetary unit (valuations, costs, and profit enter only through differences and ratios, etc.), we normalize the fresh-product valuation to $v_F = 1$ and express all monetary quantities as fractions of it.
- Normalizing $\Lambda = 1$: Given the model is scale-free in its time unit as the transition rates enter only through ratios such as μ/Λ and μ_c/ρ, so we normalize the aggregate arrival rate to $\Lambda = 1$. The per-unit rates μ, μ_c, and ρ are expressed as fractions of Λ.

Table A.1 presents the values that we use for each parameter, where we assume that each parameter follows a uniform distribution, similar to Gao and Souza (2022) and Peng et al. (2026).

Table A.1 Values of the parameters used for the robustness check.

Parameter	Support
Unit restocking cost c	$(0.35, 0.45)$
Less-fresh valuation v_L	$(0.75, 0.85)$
Household consumption rate ρ	$(0.25, 0.35)$
Household spoilage rate μ_c	$(0.25, 0.35)$
Baseline store spoilage rate μ	$(0.30, 0.40)$
Mean in-store hassle cost $\bar{h}_s$	$(0.04, 0.10)$
Customer online hassle fraction α	$(0.79, 0.89)$
Customer online hassle cost h_o	$(0.0015, 0.0115)$

- *Restocking cost c*: Sanders (2024) reports gross margins of 47–65% for artisanal bread sold by a U.S. grocery chain, implying replacement costs of 35–53% of the retail price. Thus, we select $c \in [0.35, 0.45] \cdot v_F$.
- *Less-fresh valuation* v_L: We use relative willingness to pay as a proxy for the relative value of less-fresh products. Paterson and Clark (2020) report mean bids for near-end-of-code 2% milk that are approximately 73–84% of those for fresh milk. For produces, Collart et al. (2022) report mean bids of \$1.43 per pound for less fresh sweet potatoes versus \$1.67 per pound for fresh sweet potatoes, implying a valuation ratio of approximately 0.86. These freshness-related comparisons motivate $v_L \in [0.75, 0.85] \cdot v_F$.
- *Consumption rate* ρ: Wucher et al. (2020) find that households commonly finish fruit and vegetables within three to five days. Since we assume the customer household consumption time is exponentially distributed, then a three to five day mean consumption time implies a consumption rates of of approximately 0.25–0.33. Thus, we assume $\rho \in [0.25, 0.35] \cdot \Lambda$.
- *Household spoilage rate* μ_c: Nicosia et al. (2022) find that grocery remains suitable for consumption for six to seven days under simulated domestic use and storage. In our model, a fresh item passes through two tandem spoilage stages, each with rate μ_c, so its mean lifetime is $1/\mu_c + 1/\mu_c = 2/\mu_c$. Then, an average spoiling time of 6-7 days gives the household spoilage rate approximately 0.286–0.333; Thus, we use the surrounding range $\mu_c \in [0.25, 0.35] \cdot \Lambda$.
- *Store spoilage rate* μ: Caponigro et al. (2010) report that ready-to-eat product sampled from retail stores have remaining shelf lives with a mode of five days. Since the product in-store deterioration also occurs through two successive stages but with a different rate μ, the mean lifetime is $2/\mu$, which yields $\mu \in [0.30, 0.40] \cdot \Lambda$.
- *Mean in-store hassle* $\bar{h}_s$: Grocery-store travel averages about 15 minutes one way (Hamrick and Hopkins 2012), motivating a positive travel and shopping burden that we allocate across the purchased basket; we assume a modest per-item mean hassle $\bar{h}_s \in [0.04, 0.10] \cdot v_F$.
- *BOPS hassle fraction* α *and online cost* h_o: The multiplier α captures that BOPS retains pickup travel while removing in-store search and checkout effort; we assume $\alpha \in [0.79, 0.89]$,

representing a 11–21% reduction in this component of hassle before adding online-ordering effort. The term h_o captures the latter effort. At $\alpha = 0.89$, the mean-hassle savings threshold $(1-\alpha)\bar{h}_s$ have an upper bound of $0.011 v_F$. We therefore use the fixed proposal range $h_o \in [0.0015, 0.0115] \cdot v_F$.

Data generation. We independently draw each of the system parameters $(v_L, \mu, \mu_c, \rho, c, \bar{h}_s, \alpha, h_o)$ from its specified continuous uniform distribution in Table A.1, with $v_F = \Lambda = 1$. Within each system instance, individual in-store hassle costs satisfy $h_s \sim U[\bar{h}_s - 0.02, \bar{h}_s + 0.02]$. A consumer with in-store hassle cost h_s incurs BOPS hassle cost $h_b = \alpha h_s + h_o$. For each candidate instance, we compute the profit-maximizing equilibria using Lemmas A4.1 and A4.2 for the conventional and omnichannel models, respectively. We retain the instance only if both models have strictly positive optimized baseline profits. Otherwise, we discard the instance and redraw a new set of parameters. We repeat this procedure until 200,000 eligible instances are obtained.

Store-waste comparison For each eligible instance, we examine reduced store spoilage rates $0 < \mu' < \mu$, holding all other parameters fixed and comparing each model's store waste with its own baseline at μ. We look for either of two responses when the spoilage rate is reduced to μ': BOPS store waste increases while conventional store waste decreases, or BOPS store waste decreases while conventional store waste increases.

To show the influence of the in-store hassle on these store-waste responses, we divide the range of $\bar{h}_s$ into three equal-width intervals. Within each interval, let $N^+_{(2)}$ denote the number of retained instances with BOPS store waste increasing while conventional store waste decreases. Similarly, let $N^-_{(2)}$ denote the number with BOPS store waste decreasing while conventional store waste increases. Their conditional probabilities are

$$\widehat{P}^+_{(2)} = \frac{N^+_{(2)}}{N^+_{(2)} + N^-_{(2)}} \text{ and } \widehat{P}^-_{(2)} = \frac{N^-_{(2)}}{N^+_{(2)} + N^-_{(2)}} = 1 - \widehat{P}^+_{(2)}. \tag{A.4}$$

Here, $M = N^+_{(2)} + N^-_{(2)}$ is the number of instances within the interval exhibiting exactly one of the two opposing store-waste responses. Thus, $\widehat{P}^+_{(2)}$ measures the relative frequency of BOPS store waste increasing while conventional store waste decreases among these retained instances. A similar interpretation applies to $\widehat{P}^-_{(2)}$.

Profit-gain comparison. For the same retained instances at a reduced rate μ' for the store-waste comparison, we also compare the profit gains associated with reductions that produce opposite store-waste responses, where $\Delta_{\pi^B} = \pi^B(\mu') - \pi^B(\mu)$ and $\Delta_{\pi^C} = \pi^C(\mu') - \pi^C(\mu)$. Similarly, to show the impact of $\bar{h}_s$ on these profit-gain changes, we follow the same three intervals in store-waste comparison and define $n^<_{(2)}$ as the number of instances with $\Delta_{\pi^B} < \Delta_{\pi^C}$ and $n^>_{(2)}$ as that with $\Delta_{\pi^B} > \Delta_{\pi^C}$. Then, we also define the conditional probabilities as follows:

$$\widehat{P}^<_{(2)} = \frac{n^<_{(2)}}{n^<_{(2)} + n^>_{(2)}}, \text{ and } \widehat{P}^>_{(2)} = \frac{n^>_{(2)}}{n^<_{(2)} + n^>_{(2)}} = 1 - P^<_{(2)}. \tag{A.5}$$

Results and interpretation. In Figure A.1, we observe that as $\bar{h}_s$ increases, the conditional probability $P^+_{(2)}$ decreases from 30.45% to 1.43%, while the probability that BOPS store waste decreases while conventional store waste increases ($P^+_{(2)}$) rises from 69.55% to 98.57%. This trend implies that (1) a reduction in the store spoilage rate can have opposite effects on waste in the conventional and omnichannel models, and (2) conditional on waste outcomes differing between the models, a spoilage-rate reduction is more likely to increase omnichannel store waste while decreasing conventional store waste when the in-store hassle cost is low, whereas the reverse pattern becomes more prevalent as the in-store hassle cost increases. These patterns support the robustness of the store-waste insights in Proposition 2 under heterogeneous hassle costs.

A similar logic follows for the profit gain comparison. Figure A.2 suggests that when reducing the spoilage rate, there are more instances with the gain in profit for the omnichannel model smaller than that for the conventional model when $\bar{h}_s$ is small. But with $\bar{h}_s$ rising, the instances with a larger profit gain in omnichannel model starts to dominate. This also coincides with the insights about profit change in Proposition 2: reducing the spoilage rate is more favorable in the conventional model (profit-wise) when the store is convenient (with a small in-store hassle cost); when the store is inconvenient, the mechanism is different and the comparison can reverse.

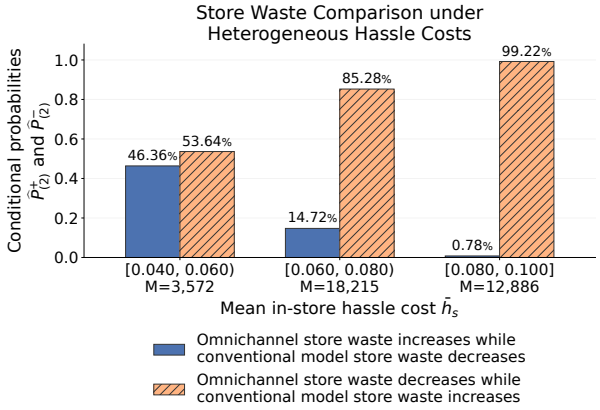


Figure A.1 Store waste comparison

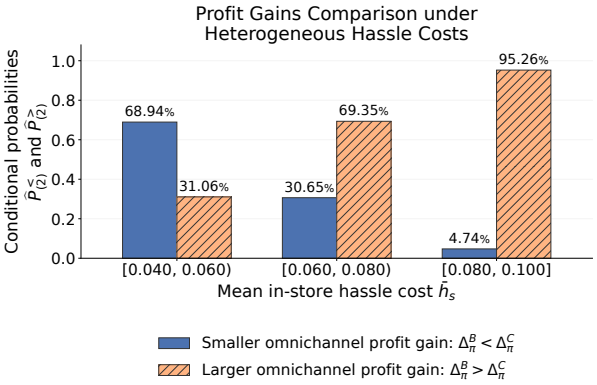


Figure A.2 Profit gain comparison

A.4.3. Impact of Retail Waste Tax

Similar to the main model, we define $W_{s,T}$, $W_{c,T}$, $W_{t,T}$ with the corresponding $\Delta_{W^B}, \Delta_{W^C}$ for the changes of the total waste for the conventional and omnichannel models respectively. The following proposition shows that Proposition 3 is robust when the in-store hassle cost is heterogeneous.

PROPOSITION A4.1. *In both the conventional and omnichannel models with heterogeneous hassle cost, imposing tax weakly decreases the store waste (i.e., $W^C_{s,T} \le W^C_{s,0}$ and $W^B_{s,T} \le W^B_{s,0}$) and weakly increases the customer waste (i.e., $W^C_{c,T} \ge W^C_{c,0}$ and $W^B_{c,T} \ge W^B_{c,0}$).*

Proposition A4.1 shows that our Proposition 3 continues to hold when customer in-store hassle cost is heterogeneous. To demonstrate that Proposition 4 is robust, we first show an analytical result about the sufficient condition for $\Delta_{W^C} \geq 0$ and $\Delta_{W^B} \geq 0$.

PROPOSITION A4.2. *In both the conventional and omnichannel models under the heterogeneous hassle cost setting, imposing tax weakly increases total waste when μ_c is large enough.*

Proposition A4.2 demonstrates that our insights in Proposition 4 is robust under the heterogeneous hassle cost setting: imposing tax can increase total waste especially when the customer spoilage rate is high. For the rest of the insights derived from Proposition 4, we show the results through numerical experiments.

Numerical Robustness of Proposition 4 Following the parameter setup and data-generation procedure in Section A.4.2, we independently generate a separate sample of 200,000 eligible instances to examine the comparison of tax-induced total-waste changes in Proposition 4. We require strictly positive optimized profits in both models at the tax level. Candidates failing these requirements are discarded and redrawn.

Total waste comparison For each eligible instance, we recompute the profit-maximizing equilibria at some tax levels for the conventional and omnichannel models, holding all system parameters fixed. At a tax level $T > 0$, we compare the change in total waste relative to each model's zero-tax baseline: $\Delta_{W^B}(T) = W^B_{t,T} - W^B_{t,0}$ and $\Delta_{W^C}(T) = W^C_{t,T} - W^C_{t,0}$, where total waste is the sum of store and customer waste.

To demonstrate the influence of online ordering hassle h_o, we divide its range into three equal-width intervals. Within each interval, let $N^+_{(4)}$ denote the number of instances with a larger tax-induced positive total-waste increase in the omnichannel model than that in the conventional model ($\Delta_{W^B}(T) > \Delta_{W^C}(T) \geq 0$), and let $N^-_{(4)}$ denote the number with a larger positive increase under the conventional model ($0 \leq \Delta_{W^B}(T) < \Delta_{W^C}(T)$).

$$\widehat{P}^+_{(4)} = \frac{N^+_{(4)}}{N^+_{(4)} + N^-_{(4)}} \text{ and } \widehat{P}^-_{(4)} = \frac{N^-_{(4)}}{N^+_{(4)} + N^-_{(4)}} = 1 - \widehat{P}^+_{(4)}. \tag{A.6}$$

Results and interpretation. In Figure A.3, we observe that as h_o increases, the conditional probability of a larger tax-induced total-waste increase in the omnichannel model ($\widehat{P}^+_{(4)}$) rises from 48.52% to 94.76%, while the probability of a larger increase under the conventional model ($\widehat{P}^-_{(4)}$) decreases from 51.48% to 5.24%. This trend implies that a larger total-waste increase under the conventional model is relatively more likely when online ordering hassle is small, whereas a larger increase in the omnichannel model becomes more prevalent as online ordering hassle increases. These patterns support the robustness of the total-waste insights in Proposition 4 under heterogeneous hassle costs: the tax's unintended consequences are more likely to be greater in the omnichannel model when online ordering is less convenient.

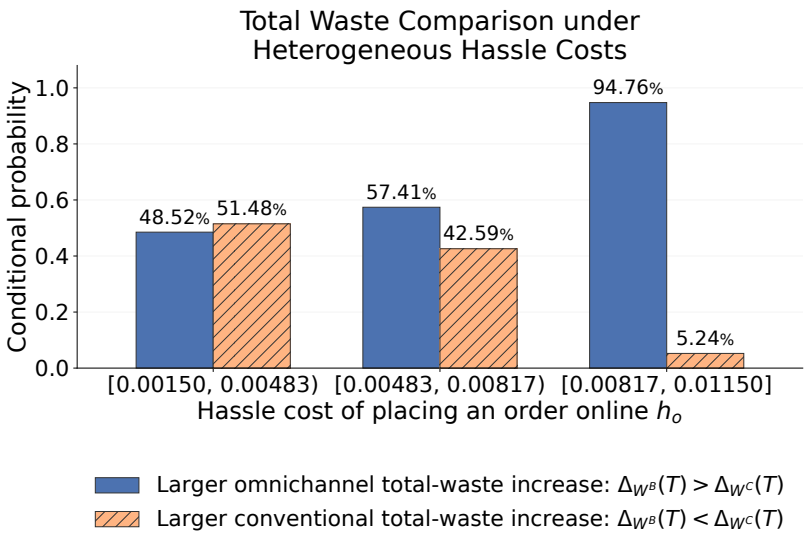


Figure A.3 Tax-induced total waste comparison

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Supplementary Material

This supplementary material contains the proofs of the results in Sections A.2, A.3, and A.4 of the online appendix. We retain the notation and the customer and grocer selection conventions stated in those sections.

S1. Proofs of Results in Section A.2

Throughout this subsection, $\Lambda, \mu, \rho, \mu_c > 0$, $V_F > V_L \geq 0$, $h_s > 0$, $c \geq 0$, $0 < \alpha < 1$, $h_o \geq 0$, and $\psi \in [0,1]$. The expected household valuations V_F and V_L retain their definitions in Equations (4) and (3), and $h_b = \alpha h_s + h_o$.

Proof of Lemma A2.1. Because the price is nonnegative, we consider two operating regimes: (i) customers purchase only fresh products, so $V_L < p \leq V_F$, and (ii) customers purchase both fresh and less-fresh products, so $0 \leq p \leq V_L$. At a price above V_F, no customer purchases a product and positive profit is impossible.

(i) $V_L < p \leq V_F$. When customers purchase only fresh products, the demand rates are $\lambda_F = \Lambda$ and $\lambda_L = 0$. Substituting $\lambda_L = 0$ into the stationary distribution in Section 6.1 gives $\theta_{(F,F)} = \frac{2\mu}{8\mu} = \frac{1}{4}, \theta_{(F,L)} = \frac{4\mu}{8\mu} = \frac{1}{2}, \theta_{(L,L)} = \frac{2\mu}{8\mu} = \frac{1}{4}$. An in-store customer purchases a fresh unit in either of the first two states and purchases nothing in the third state. Her utility is u_s and participation holds when

$$u_s = -h_s + \left(\frac{1}{4} + \frac{1}{2}\right)(V_F - p) = -h_s + \frac{3}{4}(V_F - p) \geq 0 \iff p \leq V_F - \frac{4}{3}h_s.$$

Fresh units are sold at rate $3\Lambda/4$, and the store-waste rate is $\mu\theta_{(F,L)} + 2\mu\theta_{(L,L)} = \frac{\mu}{2} + \frac{\mu}{2} = \mu$. The profit function becomes $\pi(p) = \frac{3\Lambda}{4}(p - c) - c\mu$, which is strictly increasing in p. The highest feasible price and its associated profit are thus $p^C = V_F - \frac{4}{3}h_s$ and $\pi^C = \frac{3\Lambda}{4}\left(V_F - \frac{4}{3}h_s - c\right) - c\mu$.

This regime is internally consistent if and only if $h_s < \frac{3}{4}(V_F - V_L)$.

(ii) $0 \leq p \leq V_L$. When customers purchase both fresh and less-fresh products, the demand rates are $\lambda_F = \lambda_L = \Lambda$. The stationary probabilities are $\theta_{(F,F)} = \frac{\Lambda + 2\mu}{3\Lambda + 8\mu}, \theta_{(F,L)} = \frac{2(\Lambda + 2\mu)}{3\Lambda + 8\mu}, \theta_{(L,L)} = \frac{2\mu}{3\Lambda + 8\mu}$. An in-store customer purchases fresh whenever at least one fresh unit is available. Substituting these probabilities into her utility u_s and the participation condition gives

$$\begin{aligned} u_s &= -h_s + \frac{3(\Lambda + 2\mu)}{3\Lambda + 8\mu}(V_F - p) + \frac{2\mu}{3\Lambda + 8\mu}(V_L - p) = -h_s + \frac{3(\Lambda + 2\mu)V_F + 2\mu V_L}{3\Lambda + 8\mu} - p \geq 0 \\ &\iff p \leq \frac{3(\Lambda + 2\mu)V_F + 2\mu V_L}{3\Lambda + 8\mu} - h_s. \end{aligned}$$

Every participating customer purchases one unit, so the sales rate is Λ. The store-waste rate is

$$\mu\theta_{(F,L)} + 2\mu\theta_{(L,L)} = \frac{2\mu(\Lambda + 2\mu)}{3\Lambda + 8\mu} + \frac{4\mu^2}{3\Lambda + 8\mu} = \frac{2\mu(\Lambda + 4\mu)}{3\Lambda + 8\mu}.$$

The profit function becomes $\pi(p) = \Lambda(p-c) - \frac{2c\mu(\Lambda+4\mu)}{3\Lambda+8\mu}$, which is strictly increasing in p. The grocer charges the highest price and the resulting profit is

$$p^C = \min\left\{V_L, \frac{3(\Lambda+2\mu)V_F + 2\mu V_L}{3\Lambda+8\mu} - h_s\right\} \text{ and } \pi^C = \Lambda(p^C - c) - \frac{2c\mu(\Lambda+4\mu)}{3\Lambda+8\mu}.$$

The acceptance bound binds when $h_s \le \frac{3(\Lambda+2\mu)(V_F-V_L)}{3\Lambda+8\mu}$, giving $p^C = V_L$. Otherwise, the participation bound binds. A nonnegative candidate price exists if and only if $h_s \le \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu}$.

Comparing the two feasible regimes. We first compare their feasibility ranges. Since $\frac{3(\Lambda+2\mu)}{3\Lambda+8\mu} > \frac{3}{4}$, we have $\frac{3}{4}(V_F - V_L) < \frac{3(\Lambda+2\mu)(V_F-V_L)}{3\Lambda+8\mu} \le \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu}$. Consequently, whenever the fresh-only regime is feasible, the full-sale regime is also feasible and its candidate price is V_L. We therefore compare $\pi_1^C = \frac{3\Lambda}{4}\left(V_F - \frac{4}{3}h_s - c\right) - c\mu$ with $\pi_2^C = \Lambda(V_L - c) - \frac{2c\mu(\Lambda+4\mu)}{3\Lambda+8\mu}$ to find the threshold of h_s which makes fresh only sales weakly more profitable:

$$\begin{aligned}\pi_1^C - \pi_2^C &= \Lambda\left(\frac{3V_F - 4V_L}{4} - h_s\right) + c\left[\frac{\Lambda}{4} - \mu + \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}\right] = \Lambda\left(\frac{3V_F-4V_L}{4} - h_s\right) + c\left[\frac{\Lambda}{4} - \frac{\Lambda\mu}{3\Lambda+8\mu}\right] \\ &= \Lambda\left[\frac{3V_F-4V_L}{4} + \frac{c(3\Lambda+4\mu)}{4(3\Lambda+8\mu)} - h_s\right] \ge 0 \iff h_s \le \frac{3V_F-4V_L}{4} + \frac{c(3\Lambda+4\mu)}{4(3\Lambda+8\mu)}.\end{aligned}$$

Thus, selecting fresh-only sales when the two feasible profits tie gives the threshold

$$\mathcal{H}_s^m = \min\left\{\frac{3}{4}(V_F - V_L), \frac{3V_F-4V_L}{4} + \frac{c(3\Lambda+4\mu)}{4(3\Lambda+8\mu)}\right\}.$$

For completeness, the weak inequality in the lemma does not assign an infeasible fresh-only outcome at $h_s = 3(V_F - V_L)/4$ under the maintained positive-profit condition. At that boundary, the full-sale candidate price is V_L. Positive profit then requires $c < V_L$. Hence

$$\frac{3V_F-4V_L}{4} + \frac{c(3\Lambda+4\mu)}{4(3\Lambda+8\mu)} - \frac{3}{4}(V_F - V_L) = \frac{1}{4}\left[-V_L + \frac{c(3\Lambda+4\mu)}{3\Lambda+8\mu}\right] < 0.$$

Thus, the profit-comparison threshold is strictly below the feasibility boundary in that situation, and the boundary is correctly assigned to full sales.

Positive-profit conditions. Finally, we establish the condition for strictly positive optimal profit. In this calculation, π_1^C denotes profit at the fresh-only candidate price and π_2^C denotes profit at the full-sale candidate price $\min\{V_L, [3(\Lambda+2\mu)V_F + 2\mu V_L]/(3\Lambda+8\mu) - h_s\}$.

In the fresh-only regime,

$$\pi_1^C > 0 \iff \frac{3\Lambda}{4}\left(V_F - \frac{4}{3}h_s\right) > c\left(\frac{3\Lambda}{4} + \mu\right) \iff c < \frac{3\Lambda(V_F - \frac{4}{3}h_s)}{3\Lambda+4\mu}.$$

In the full-sale regime,

$$\begin{aligned}\pi_2^C > 0 &\iff \Lambda \min\left\{V_L, \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu} - h_s\right\} > c\left[\Lambda + \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}\right] \\ &\iff c < \frac{\Lambda}{\Lambda + \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}} \min\left\{V_L, \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu} - h_s\right\}.\end{aligned}$$

When $h_s < 3(V_F - V_L)/4$, both regimes are feasible, so strictly positive optimal profit is equivalent to at least one of these two inequalities. When $h_s \geq 3(V_F - V_L)/4$, only the full-sale regime can generate sales at an internally consistent candidate price. We verify that the first displayed cost bound does not enlarge the positive-profit region in this case. If the full-sale candidate price is V_L, then $V_F - 4h_s/3 \leq V_L$, and $\frac{3\Lambda(V_F - \frac{4}{3}h_s)}{3\Lambda+4\mu} \leq \frac{3\Lambda V_L}{3\Lambda+4\mu} \leq \frac{\Lambda(3\Lambda+8\mu)V_L}{(3\Lambda+4\mu)(\Lambda+2\mu)}$, where the second inequality follows from $3(\Lambda+2\mu) \leq 3\Lambda+8\mu$. If the participation bound determines the full-sale price, the difference between the second and first cost bounds equals

$$\frac{\Lambda(3\Lambda+8\mu)}{(3\Lambda+4\mu)(\Lambda+2\mu)}\left[\frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu} - h_s\right] - \frac{\Lambda(3V_F-4h_s)}{3\Lambda+4\mu} \qquad = \frac{\Lambda(2\mu V_L+\Lambda h_s)}{(3\Lambda+4\mu)(\Lambda+2\mu)} > 0.$$

Thus, in the region where fresh-only sales are infeasible, the maximum of the two cost bounds is exactly the full-sale bound. For hassle above the full-sale participation limit, no nonnegative price can produce a positive-profit participating outcome. We have therefore established that strictly positive optimal profit holds if and only if $h_s \leq \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu}$ and

$$c < \max\left\{\frac{3\Lambda(V_F-\frac{4}{3}h_s)}{3\Lambda+4\mu}, \frac{\Lambda}{\Lambda+\frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}}\min\left\{V_L, \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu} - h_s\right\}\right\}.$$

□

Proof of Lemma A2.2. Maintain the positive conventional-profit condition in Lemma A2.1. We first derive the highest feasible price when every customer uses BOPS, and then compare the corresponding profit with the conventional optimum. The grocer can always obtain the conventional optimum by not offering BOPS.

BOPS stationary distribution and profit. When every customer uses BOPS, the purchase rates are $\lambda_o = \Lambda$ and $\lambda_{F,s} = \lambda_{L,s} = 0$. Substituting these rates into the stationary probabilities gives $\theta_{(F,F)} = \frac{[\mu+(1-\psi)\Lambda](\Lambda+2\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}, \theta_{(F,L)} = \frac{2\mu(\Lambda+2\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}, \theta_{(L,L)} = \frac{2\mu^2}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}$. The probability that a BOPS customer receives a less-fresh unit is

$$\theta_{(L,L)} + (1-\psi)\theta_{(F,L)} = \frac{2\mu^2+2(1-\psi)\mu(\Lambda+2\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} = \frac{2\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}.$$

Her utility is consequently $u_b = -h_b + V_F - \frac{2\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F - V_L) - p$. Every arrival purchases one unit. The sales rate is Λ, and the store-waste rate is

$$W_s^B = \mu\theta_{(F,L)} + 2\mu\theta_{(L,L)} = \frac{2\mu^2(\Lambda+2\mu)+4\mu^3}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} = \frac{2\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}.$$

Therefore, $\pi(p) = \Lambda(p-c) - \frac{2c\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}$, which is strictly increasing in p. The grocer charges the highest price satisfying both $u_b \geq 0$ and $u_b \geq u_s$.

Participation and channel preference. The participation condition $u_b \geq 0$ is equivalent to $p \leq V_F - \frac{2\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F - V_L) - h_b$. For channel preference, we consider the same two price regions as in the proof for the main model.

If $V_L < p \leq V_F$, an in-store customer accepts only a fresh unit, so $u_s = -h_s + (\theta_{(F,F)} + \theta_{(F,L)})(V_F - p)$. Since $\theta_{(L,L)} > 0$, the inequality $u_b \geq u_s$ is equivalent to

$$\begin{aligned} u_b - u_s &= h_s - h_b - (1-\psi)\theta_{(F,L)}(V_F - V_L) + \theta_{(L,L)}(V_L - p) \geq 0 \\ &\iff p \leq V_L - \frac{(1-\psi)(\Lambda+2\mu)}{\mu}(V_F - V_L) + \frac{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}{2\mu^2}(h_s - h_b). \end{aligned}$$

If $p \leq V_L$, an in-store customer purchases in every shelf state, taking the fresh unit when one is available. In this price region, $u_s = -h_s + (\theta_{(F,F)} + \theta_{(F,L)})V_F + \theta_{(L,L)}V_L - p$, and hence

$$u_b - u_s = h_s - h_b - (1-\psi)\theta_{(F,L)}(V_F - V_L) \iff h_s - h_b \geq \frac{2(1-\psi)\mu(\Lambda+2\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F - V_L).$$

This is also necessary in the region $p > V_L$, since the additional term $\theta_{(L,L)}(V_L - p)$ is then negative. Using $h_b = \alpha h_s + h_o$, define the feasibility threshold

$$\overline{h}_o^{\,m}(\psi) = (1-\alpha)h_s - \frac{2(1-\psi)\mu(\Lambda+2\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F - V_L).$$

There is no participating all-BOPS equilibrium when $h_o > \overline{h}_o^{\,m}(\psi)$, whatever price the grocer charges. When $h_o \leq \overline{h}_o^{\,m}(\psi)$, the channel-preference price bound displayed above is at least V_L. It then applies to both price regions: every price below V_L satisfies channel preference, and the displayed upper bound governs prices above V_L. Consequently, whenever the all-BOPS candidate is feasible, its highest price is

$$\widehat{p}^{\,B} = \min\left\{ V_F - \frac{2\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F - V_L) - h_b, \right. \\ \left. V_L - \frac{(1-\psi)(\Lambda+2\mu)}{\mu}(V_F - V_L) + \frac{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}{2\mu^2}(h_s - h_b) \right\},$$

and its profit is $\widehat{\pi}^B = \Lambda(\widehat{p}^{\,B} - c) - \frac{2c\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}$. We only compare this candidate when its price is nonnegative. Both price bounds are strictly decreasing in h_o, so the highest feasible all-BOPS price and its associated profit are strictly decreasing in h_o on their feasible range.

Case (i): $0 < h_s \leq \mathcal{H}_s^m$. The conventional model is in fresh only regime with $p^C = V_F - \frac{4}{3}h_s$, and $\pi^C = \frac{3\Lambda}{4}\left(V_F - \frac{4}{3}h_s - c\right) - c\mu$. The inequality $\widehat{\pi}^B > \pi^C$ is equivalent to

$$\widehat{p}^{\,B} > \frac{3V_F - 4h_s + c}{4} - \frac{c\mu}{\Lambda} + \frac{2c\mu^2(\Lambda+4\mu)}{\Lambda\{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2\}}.$$

Because $\widehat{p}^B$ is the minimum of the participation and channel-preference price bounds, each of those bounds must exceed the right-hand side. The participation bound gives $h_o < h_{o1}^m(\psi)$, where

$$h_{o1}^m(\psi) = V_F - \frac{2\mu[(1-\psi)\Lambda + (3-2\psi)\mu]}{[3\mu + (1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F - V_L) - \alpha h_s - \frac{3V_F - 4h_s + c}{4} + \frac{c\mu}{\Lambda} - \frac{2c\mu^2(\Lambda+4\mu)}{\Lambda\{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2\}}.$$

For the channel-preference bound, we have $h_o < h_{o2}^m(\psi)$, where

$$h_{o2}^m(\psi) = (1-\alpha)h_s - \frac{2(1-\psi)\mu(\Lambda+2\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F - V_L) + \frac{2\mu^2}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}\left[V_L - \frac{3V_F - 4h_s + c}{4} + \frac{c\mu}{\Lambda} - \frac{2c\mu^2(\Lambda+4\mu)}{\Lambda\{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2\}}\right].$$

Thus a strictly more profitable all-BOPS outcome exists exactly when all three conditions hold: $h_o \le \overline{h}_o^{\,m}(\psi), h_o < h_{o1}^m(\psi), h_o < h_{o2}^m(\psi)$. Define $\mathcal{H}_o^m(\psi) = \min\{\overline{h}_o^{\,m}(\psi), h_{o1}^m(\psi), h_{o2}^m(\psi)\}$. If $h_o < \mathcal{H}_o^m(\psi)$, the three required conditions hold, and the stated BOPS price and profit are strictly better than the conventional optimum. If $h_o > \mathcal{H}_o^m(\psi)$, either all-BOPS purchases are infeasible or their highest feasible price yields a profit below the conventional optimum. The grocer then does not offer BOPS.

Case (ii): $h_s > \mathcal{H}_s^m$. The equilibrium of the conventional model is in full sale regime. Then, we have $p^C = \min\left\{V_L, \frac{3(\Lambda+2\mu)V_F + 2\mu V_L}{3\Lambda+8\mu} - h_s\right\}$ and $\pi^C = \Lambda(p^C - c) - \frac{2c\mu(\Lambda+4\mu)}{3\Lambda+8\mu}$. Suppose $h_o < \overline{h}_o^{\,m}(\psi)$. The channel-feasibility inequality is then strict. The channel-preference price bound is strictly above V_L. For the participation bound, the same inequality implies

$$V_F - \frac{2\mu[(1-\psi)\Lambda + (3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F - V_L) - h_b > V_F - \frac{2\mu^2}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F - V_L) - h_s.$$

The stationary probability of an all-less-fresh shelf under BOPS satisfies

$$\frac{2\mu}{3\Lambda+8\mu} - \frac{2\mu^2}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} = \frac{2(1-\psi)\Lambda\mu(\Lambda+2\mu)}{(3\Lambda+8\mu)\{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2\}} \ge 0.$$

It follows that the participation price bound is strictly greater than

$$V_F - \frac{2\mu}{3\Lambda+8\mu}(V_F - V_L) - h_s = \frac{3(\Lambda+2\mu)V_F + 2\mu V_L}{3\Lambda+8\mu} - h_s.$$

Both BOPS price bounds therefore strictly exceed the respective conventional bounds. Taking their minima yields $\widehat{p}^B > p^C$. BOPS store-waste rate is no greater than the conventional full-sale rate:

$$\frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu} - \frac{2\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} = \frac{2(1-\psi)\Lambda\mu(\Lambda+2\mu)(\Lambda+4\mu)}{(3\Lambda+8\mu)\{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2\}} \ge 0.$$

Combining the price and store-waste comparisons gives

$$\widehat{\pi}^B - \pi^C = \Lambda(\widehat{p}^B - p^C) + c\left[\frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu} - \frac{2\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}\right] > 0.$$

Thus BOPS is strictly profitable throughout the strict feasibility range. When $h_o > \overline{h}_o^m(\psi)$, no participating all-BOPS outcome is feasible. Accordingly, define $\mathcal{H}_o^m(\psi) = \overline{h}_o^m(\psi)$ in this case and The required threshold is

$$\mathcal{H}_o^m(\psi) = \begin{cases} \min\{\overline{h}_o^m(\psi), h_{o1}^m(\psi), h_{o2}^m(\psi)\}, & 0 < h_s \le \mathcal{H}_s^m, \\ \overline{h}_o^m(\psi), & h_s > \mathcal{H}_s^m. \end{cases} \qquad \square$$

Proof of Proposition A2.1. Fix the initial store spoilage rate $\mu > 0$, and set

$$\mathcal{H}_s^m = \mathcal{H}_s^m(\mu) = \min\left\{\frac{3}{4}(V_F - V_L), \frac{3V_F - 4V_L}{4} + \frac{c(3\Lambda + 4\mu)}{4(3\Lambda + 8\mu)}\right\}.$$

We prove the proposition for the conventional model and the omnichannel model separately.

Conventional model. Define $\mathcal{H}_s^m(x) = \min\left\{\frac{3}{4}(V_F - V_L), \frac{3V_F - 4V_L}{4} + \frac{c(3\Lambda+4x)}{4(3\Lambda+8x)}\right\}$. For $x > 0$, we have $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{3V_F-4V_L}{4} + \frac{c(3\Lambda+4x)}{4(3\Lambda+8x)}\right] = -\frac{3c\Lambda}{(3\Lambda+8x)^2} \le 0$ and $\frac{3}{4}(V_F - V_L)$ is independent of x. Then, $\mathcal{H}_s^m(x)$ is non-increasing in x, and $\mathcal{H}_s^m(\mu') \ge \mathcal{H}_s^m(\mu)$ for $0 < \mu' < \mu$.

Since positive conventional profit is retained at every lower spoilage rate, then we can apply Lemma A2.1, which implies that the conventional grocer sells only fresh units when $h_s \le \mathcal{H}_s^m(x)$ and sells both freshness levels when $h_s > \mathcal{H}_s^m(x)$. We discuss the two strict cases.

Case (i): $h_s < \mathcal{H}_s^m(\mu)$. Since $h_s < \mathcal{H}_s^m(\mu) \le \mathcal{H}_s^m(\mu')$, the conventional grocer sells only fresh products at both spoilage rates. In this regime, the stationary probabilities are $\theta_{(F,F)} = \frac{1}{4}, \theta_{(F,L)} = \frac{1}{2}, \theta_{(L,L)} = \frac{1}{4}$. Then, $W_s^C(\mu) = \mu\frac{1}{2} + 2\mu\frac{1}{4} = \mu, W_s^C(\mu') = \mu'$. Since $\mu' < \mu$, store waste decreases.

Case (ii): $h_s > \mathcal{H}_s^m(\mu)$. The conventional grocer initially sells both freshness levels. Substitution of the full-sale stationary probabilities gives $W_s^C(\mu) = \mu\frac{2(\Lambda+2\mu)}{3\Lambda+8\mu} + 2\mu\frac{2\mu}{3\Lambda+8\mu} = \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}$. If $h_s > \mathcal{H}_s^m(\mu')$, the grocer still sells both freshness levels. The positive derivative calculated above implies $W_s^C(\mu') = \frac{2\mu'(\Lambda+4\mu')}{3\Lambda+8\mu'} < \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu} = W_s^C(\mu)$. If instead $h_s \le \mathcal{H}_s^m(\mu')$, the lower-rate optimum sells only fresh units, and $W_s^C(\mu') = \mu'$. Therefore, $W_s^C(\mu') \ge W_s^C(\mu)$ if and only if $\mu' \ge \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}$.

To obtain the strict switching interval in the statement, we next solve $h_s < \mathcal{H}_s^m(\mu')$. Since $h_s > \mathcal{H}_s^m(\mu)$, then $4h_s - 3V_F + 4V_L > \frac{c(3\Lambda+4\mu)}{3\Lambda+8\mu} \ge \frac{c}{2}$, which implies $8(4h_s - 3V_F + 4V_L) - 4c > 0$. Then

$$h_s < \mathcal{H}_s^m(\mu') \iff 4h_s - 3V_F + 4V_L < \frac{c(3\Lambda + 4\mu')}{3\Lambda + 8\mu'} \iff \mu' < \frac{3\Lambda(3V_F - 4V_L - 4h_s + c)}{4(8h_s - 6V_F + 8V_L - c)}.$$

Define $\mathcal{M}_{c1}^m = \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}$ and $\mathcal{M}_{c2}^m = \frac{3\Lambda(3V_F-4V_L-4h_s+c)}{4(8h_s-6V_F+8V_L-c)}$. Then, $\mu' \in (\mathcal{M}_{c1}^m, \mathcal{M}_{c2}^m)$ ensures both the strict switch to fresh-only sales and the strict increase in store waste.

Omnichannel model. For each $x > 0$, let $\mathcal{H}_o^m(x, \psi)$ denote the adoption threshold in the proof of Lemma A2.2, evaluated at spoilage rate x. When BOPS is selected, all customers purchase online. Its store-waste rate is $\frac{2x^2(\Lambda+4x)}{[3x+(1-\psi)\Lambda](\Lambda+2x)+2x^2}$. Direct differentiation gives

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{2x^2(\Lambda+4x)}{[3x+(1-\psi)\Lambda](\Lambda+2x)+2x^2} = \frac{4(1-\psi)\Lambda^3 x + (34-28\psi)\Lambda^2 x^2 + (80-32\psi)\Lambda x^3 + 64x^4}{\left([3x+(1-\psi)\Lambda](\Lambda+2x)+2x^2\right)^2} > 0.$$

Every numerator term is nonnegative for $0 \le \psi \le 1$, and the final term is strictly positive, and $\frac{2x^2(\Lambda+4x)}{[3x+(1-\psi)\Lambda](\Lambda+2x)+2x^2} \le \frac{2x(\Lambda+4x)}{3\Lambda+8x} < x$. These comparisons show that store waste decreases when both outcomes use BOPS, and that a switch from fresh-only sales to BOPS also decreases store waste.

Case (i): $h_s < \mathcal{H}_s^m(\mu)$. The conventional outcome is fresh-only at both μ and μ'. Thus, each omnichannel outcome either uses BOPS or coincides with that fresh-only conventional outcome. If BOPS is used at neither rate, waste decreases from μ to μ'. If BOPS is used at both rates, the positive derivative above proves that waste decreases. If BOPS is used only at the lower rate, its new waste is less than μ', which is itself less than μ. Therefore, an increase in omnichannel store waste is possible only when BOPS is used at μ and the conventional fresh-only outcome is selected at μ'. For the fixed pair $\mu' < \mu$, define

$$\mathcal{H}_{o1}^m(\psi) = \min\{\mathcal{H}_o^m(\mu,\psi), \mathcal{H}_o^m(\mu',\psi)\} \text{ and } \mathcal{H}_{o2}^m(\psi) = \mathcal{H}_o^m(\mu,\psi) \text{ and } \mathcal{M}_b^m(\psi) = \frac{2\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}.$$

Then, for $\mathcal{H}_o^m(\mu',\psi) < h_o < \mathcal{H}_o^m(\mu,\psi)$ and $\mu' \in (\mathcal{M}_b^m(\psi), \mu)$, BOPS is used initially and is not selected at μ'. Thus, we have $W_s^B(\mu') = \mu' > \frac{2\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} = W_s^B(\mu)$. Under precisely these conditions, $\pi_\mu^B > \pi_\mu^C$ and $\pi_{\mu'}^B = \pi_{\mu'}^C$. Consequently, $\Delta_{\pi^B} = \pi_{\mu'}^B - \pi_\mu^B < \pi_{\mu'}^C - \pi_\mu^C = \Delta_{\pi^C}$.

Case (ii): $h_s \ge \mathcal{H}_s^m(\mu)$. We first define a uniform online-hassle threshold that guarantees the stated waste reduction for every $\mu' < \mu$. We will include in the same threshold the price-branch condition needed for the profit comparison on the conventional switching interval.

Recall from Lemma A2.2 that, when BOPS is selected, its two price bounds are

$$\begin{aligned} p_1(x) =& V_F - \frac{2x[(1-\psi)\Lambda + (3-2\psi)x]}{[3x+(1-\psi)\Lambda](\Lambda+2x)+2x^2}(V_F - V_L) - \alpha h_s - h_o, \\ p_2(x) =& V_L - \frac{(1-\psi)(\Lambda+2x)}{x}(V_F - V_L) + \frac{[3x+(1-\psi)\Lambda](\Lambda+2x)+2x^2}{2x^2}[(1-\alpha)h_s - h_o]. \end{aligned}$$

Define the price-branch threshold

$$h_{o3}^m(x,\psi) = h_s\left[\frac{[3x+(1-\psi)\Lambda](\Lambda+2x)+2x^2}{[3x+(1-\psi)\Lambda](\Lambda+2x)} - \alpha\right] - \frac{2x[(1-\psi)\Lambda+(3-2\psi)x]}{[3x+(1-\psi)\Lambda](\Lambda+2x)+2x^2}(V_F - V_L).$$

Subtracting the two price bounds and collecting the terms involving h_o yields

$$p_2(x) - p_1(x) = \frac{[3x+(1-\psi)\Lambda](\Lambda+2x)}{2x^2}[h_{o3}^m(x,\psi) - h_o].$$

The coefficient is strictly positive. Therefore, $p_2(x) \le p_1(x)$ if and only if $h_o \ge h_{o3}^m(x,\psi)$. Define $\mathcal{H}_{o3}^m(\psi) = \min\{\mathcal{H}_o^m(\mu,\psi), \mathcal{H}_o^m(\mu',\psi), h_{o3}^m(\mu',\psi)\}$.

Store waste. If $h_o < \mathcal{H}_{o3}^m(\psi)$, then $h_o < \mathcal{H}_o^m(\mu,\psi)$ and $h_o < \mathcal{H}_o^m(\mu',\psi)$. Hence, BOPS is selected at μ and μ'. The positive derivative of its store-waste expression gives

$$W_s^B(\mu') = \frac{2(\mu')^2(\Lambda+4\mu')}{[3\mu'+(1-\psi)\Lambda](\Lambda+2\mu')+2(\mu')^2} < \frac{2\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} = W_s^B(\mu).$$

Profit improvement. If $\mu' \in (\mathcal{M}^m_{c1}, \mathcal{M}^m_{c2})$, the equilibrium for the conventional model is full-sale at μ and fresh-only at μ'. The latter condition implies $h_s < \frac{3}{4}(V_F - V_L)$. Since $\frac{3(\Lambda+2\mu)}{3\Lambda+8\mu} > \frac{3}{4}$, it follows that $h_s < \frac{3(\Lambda+2\mu)}{3\Lambda+8\mu}(V_F - V_L)$. Thus, the initial conventional full-sale price is V_L. Define the profits of the two conventional candidates by their feasible regime: $\pi^f_x = \frac{3\Lambda}{4}\left(V_F - \frac{4}{3}h_s - c\right) - cx$ and $\pi^\ell_\mu = \Lambda(V_L - c) - c\frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}$. Here $\pi^\ell_\mu - \pi^f_\mu > 0$, and $\Delta_{\pi^C} = \pi^f_{\mu'} - \pi^\ell_\mu = c(\mu - \mu') - (\pi^\ell_\mu - \pi^f_\mu)$. The definition of $\mathcal{H}^m_{o3}(\psi)$ implies $h_o < h^m_{o3}(\mu', \psi)$, so the participation bound determines the lower-rate BOPS price: $p^B(\mu') = p_1(\mu')$. Define

$$\mathcal{H}^m_{o4}(\psi) = h^m_{o3}(\mu, \psi) + \left[\frac{c}{2} - (V_F - V_L)\right]^+ \frac{2\mu^2}{[3\mu + (1-\psi)\Lambda](\Lambda + 2\mu)} \times \left[\frac{2\mu[(1-\psi)\Lambda + (3-2\psi)\mu]}{[3\mu + (1-\psi)\Lambda](\Lambda+2\mu) + 2\mu^2} - \frac{2\mu'[(1-\psi)\Lambda + (3-2\psi)\mu']}{[3\mu' + (1-\psi)\Lambda](\Lambda+2\mu') + 2(\mu')^2}\right].$$

Here $[a]^+ = \max\{a, 0\}$. To check that the expression in square brackets is positive, differentiate the less-fresh share of BOPS purchases:

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{2x[(1-\psi)\Lambda + (3-2\psi)x]}{[3x + (1-\psi)\Lambda](\Lambda + 2x) + 2x^2} = \frac{2\Lambda\left[(1-\psi)^2\Lambda^2 + 2(1-\psi)(3-2\psi)\Lambda x + \{3 + 4(1-\psi)^2\}x^2\right]}{\left([3x + (1-\psi)\Lambda](\Lambda + 2x) + 2x^2\right)^2} > 0.$$

Therefore, $\mathcal{H}^m_{o4}(\psi) \geq h^m_{o3}(\mu, \psi)$. The condition $h_o \geq \mathcal{H}^m_{o4}(\psi)$ ensures that the channel-preference bound determines the initial BOPS price: $p^B(\mu) = p_2(\mu)$.

We now compare the profit improvements. Using the profit formula, we have

$$\begin{aligned}
\Delta_{\pi^B} - \Delta_{\pi^C} &= \pi^\ell_\mu - \pi^f_\mu + \Lambda[p_1(\mu') - p_2(\mu)] \\
&\quad - c\left[\frac{2(\mu')^2(\Lambda + 4\mu')}{[3\mu' + (1-\psi)\Lambda](\Lambda+2\mu') + 2(\mu')^2} - \frac{2\mu^2(\Lambda + 4\mu)}{[3\mu + (1-\psi)\Lambda](\Lambda+2\mu) + 2\mu^2}\right] - c(\mu - \mu') \\
&= \pi^\ell_\mu - \pi^f_\mu + \Lambda[p_1(\mu') - p_2(\mu)] \\
&\quad + \frac{c\Lambda}{2}\left[\frac{2\mu'[(1-\psi)\Lambda + (3-2\psi)\mu']}{[3\mu' + (1-\psi)\Lambda](\Lambda+2\mu') + 2(\mu')^2} - \frac{2\mu[(1-\psi)\Lambda + (3-2\psi)\mu]}{[3\mu + (1-\psi)\Lambda](\Lambda+2\mu) + 2\mu^2}\right].
\end{aligned}$$

Furthermore,

$$\begin{aligned}
p_1(\mu') - p_2(\mu) &= p_1(\mu') - p_1(\mu) + p_1(\mu) - p_2(\mu) \\
&= (V_F - V_L)\left[\frac{2\mu[(1-\psi)\Lambda + (3-2\psi)\mu]}{[3\mu + (1-\psi)\Lambda](\Lambda+2\mu) + 2\mu^2} - \frac{2\mu'[(1-\psi)\Lambda + (3-2\psi)\mu']}{[3\mu' + (1-\psi)\Lambda](\Lambda+2\mu') + 2(\mu')^2}\right] \\
&\quad + \frac{[3\mu + (1-\psi)\Lambda](\Lambda + 2\mu)}{2\mu^2}[h_o - h^m_{o3}(\mu, \psi)].
\end{aligned}$$

Substituting this expression into the profit difference gives

$$\begin{aligned}
\Delta_{\pi^B} - \Delta_{\pi^C} &= \pi^\ell_\mu - \pi^f_\mu + \Lambda\frac{[3\mu + (1-\psi)\Lambda](\Lambda + 2\mu)}{2\mu^2}[h_o - h^m_{o3}(\mu, \psi)] \\
&\quad + \Lambda\left(V_F - V_L - \frac{c}{2}\right)\left[\frac{2\mu[(1-\psi)\Lambda + (3-2\psi)\mu]}{[3\mu + (1-\psi)\Lambda](\Lambda+2\mu) + 2\mu^2} - \frac{2\mu'[(1-\psi)\Lambda + (3-2\psi)\mu']}{[3\mu' + (1-\psi)\Lambda](\Lambda+2\mu') + 2(\mu')^2}\right].
\end{aligned}$$

If $V_F - V_L \geq c/2$, the middle term is nonnegative, and $h_o \geq \mathcal{H}^m_{o4}(\psi) = h^m_{o3}(\mu, \psi)$ makes the final term nonnegative. If $V_F - V_L < c/2$, the added term in $\mathcal{H}^m_{o4}(\psi)$ ensures that the final term is at least the absolute value of the negative middle term. In either case, the last two contributions sum to a nonnegative value. Since $\pi^\ell_\mu - \pi^f_\mu > 0$, we conclude that $\Delta_{\pi^B} > \Delta_{\pi^C}$. □

Proof of Proposition A2.2. Recall the taxed profit from the model. For a fixed feasible stationary outcome, the tax enters profit only through the store-waste term, so $\frac{\partial \pi_T(p)}{\partial T} = -W_{s,T} \le 0$. Thus, taxed profit is weakly decreasing in the tax at every feasible price. The optimized profit is therefore weakly decreasing as well. To show the effects on waste, we first characterize the store-waste and customer-waste rates in each feasible sales regime.

Stationary outcomes and store-waste rates. We use superscripts f, ℓ, and b to distinguish fresh-only in-store sales, sales of both freshness levels in store, and purchases through BOPS, respectively. When customers purchase only fresh products in store, the demand rates are $\lambda_{F,s} = \Lambda$, $\lambda_{L,s} = \lambda_o = 0$. Substituting these rates into the stationary distribution gives $\theta_{(F,F)} = \frac{1}{4}, \theta_{(F,L)} = \frac{1}{2}, \theta_{(L,L)} = \frac{1}{4}$. Hence, the fresh-sales rate is $3\Lambda/4$, no less-fresh product is sold, and $W_s^f = \mu\frac{1}{2} + 2\mu\frac{1}{4} = \mu$. When customers purchase both freshness levels in store, the demand rates are $\lambda_{F,s} = \lambda_{L,s} = \Lambda$ and $\lambda_o = 0$. Thus, $\theta_{(F,F)} = \frac{\Lambda+2\mu}{3\Lambda+8\mu}, \theta_{(F,L)} = \frac{2(\Lambda+2\mu)}{3\Lambda+8\mu}, \theta_{(L,L)} = \frac{2\mu}{3\Lambda+8\mu}$. Every arriving customer purchases one unit, and $W_s^\ell = \frac{2\mu(\Lambda+2\mu)}{3\Lambda+8\mu} + \frac{4\mu^2}{3\Lambda+8\mu} = \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}$. When customers purchase through BOPS, $\lambda_o = \Lambda$ and $\lambda_{F,s} = \lambda_{L,s} = 0$. The stationary distribution in this setting is $\theta_{(F,F)} = \frac{[\mu+(1-\psi)\Lambda](\Lambda+2\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}, \theta_{(F,L)} = \frac{2\mu(\Lambda+2\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}, \theta_{(L,L)} = \frac{2\mu^2}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}$. Every arriving customer again purchases one unit. Therefore, $W_s^b = \frac{2\mu^2(\Lambda+2\mu)+4\mu^3}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} = \frac{2\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}$. The store-waste comparisons follow by subtraction: $W_s^f - W_s^\ell = \mu - \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu} = \frac{\Lambda\mu}{3\Lambda+8\mu} > 0$, and

$$W_s^\ell - W_s^b = \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu} - \frac{2\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} = \frac{2(1-\psi)\Lambda\mu(\Lambda+2\mu)(\Lambda+4\mu)}{(3\Lambda+8\mu)\{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2\}} \ge 0.$$

Consequently, $W_s^f > W_s^\ell \ge W_s^b$.

Within each regime the feasible prices do not depend on the tax, and the store-waste rate does not depend on price. Thus, maximizing profit within a regime leaves the slope in the tax equal to the negative of its store-waste rate. A regime with greater store waste loses profit faster as the tax increases. To make the implication precise, let a regime with store waste W_1 be optimal at tax T_1 and a regime with store waste W_2 be optimal at tax $T_2 > T_1$. Denote their optimized untaxed profits by $\pi_1(0)$ and $\pi_2(0)$. Optimality gives $\pi_1(0) - T_1W_1 \ge \pi_2(0) - T_1W_2$, $\pi_2(0) - T_2W_2 \ge \pi_1(0) - T_2W_1$. Adding these inequalities yields $(T_2 - T_1)(W_1 - W_2) \ge 0$. Since $T_2 > T_1$, we have $W_2 \le W_1$. Therefore, the tax cannot induce a switch to a regime with greater store waste. In the conventional model, the only possible change is from fresh-only sales to full in-store sales. In the omnichannel model, any regime changes also follow the order from fresh-only sales toward full in-store sales or BOPS, and from full in-store sales toward BOPS. If two regimes have the same store waste, the only possible distinct pair here is full in-store sales and BOPS at $\psi = 1$.

Effect on store waste. If the regime does not change, store waste does not change. If the regime changes, the preceding optimality comparison shows that the new regime has no greater store waste. Taking $T_1 = 0$ and $T_2 = T > 0$ gives $W_{s,T}^C \le W_{s,0}^C, W_{s,T}^B \le W_{s,0}^B$.

Effect on customer waste. The customer-waste rate in the fresh-only regime is $W_c^f = \frac{3\Lambda}{4}\left(\frac{\mu_c}{\rho+\mu_c}\right)^2$. The customer-waste rate in the full in-store regime is

$$W_c^\ell = \frac{3\Lambda(\Lambda+2\mu)}{3\Lambda+8\mu}\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \frac{2\Lambda\mu}{3\Lambda+8\mu}\frac{\mu_c}{\rho+\mu_c} = \Lambda\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \frac{2\Lambda\mu}{3\Lambda+8\mu}\frac{\mu_c\rho}{(\rho+\mu_c)^2}.$$

Subtracting the fresh-only rate gives $W_c^\ell - W_c^f = \frac{\Lambda}{4}\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \frac{2\Lambda\mu}{3\Lambda+8\mu}\frac{\mu_c\rho}{(\rho+\mu_c)^2} > 0$.

Under BOPS, the probability that the purchased unit is less fresh is

$$\theta_{(L,L)} + (1-\psi)\theta_{(F,L)} = \frac{2\mu[(1-\psi)\Lambda + (3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}.$$

Consequently, $W_c^b = \Lambda\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \frac{2\Lambda\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}\frac{\mu_c\rho}{(\rho+\mu_c)^2}$.. Then, we have

$$W_c^b - W_c^\ell = \frac{4(1-\psi)\Lambda\mu(\Lambda+2\mu)(\Lambda+4\mu)}{(3\Lambda+8\mu)\{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2\}}\frac{\mu_c\rho}{(\rho+\mu_c)^2} \geq 0,$$

by the fact that

$$\frac{2\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} - \frac{2\mu}{3\Lambda+8\mu} = \frac{4(1-\psi)\mu(\Lambda+2\mu)(\Lambda+4\mu)}{(3\Lambda+8\mu)\{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2\}} \geq 0.$$

Thus, customer waste is ordered as $W_c^f < W_c^\ell \leq W_c^b$, exactly opposite to the order of store waste. At $\psi = 1$, the two full-sale regimes have equal customer waste as well as equal store waste. Since a tax can only move the grocer toward a regime with no greater store waste, every possible change has no smaller customer waste. Hence, $W_{c,T}^C \geq W_{c,0}^C, W_{c,T}^B \geq W_{c,0}^B$. □

Proof of Proposition A2.3. Assume $c > 0$, $h_b < h_s$, $(1-\psi)\Lambda^2 + (1+2\psi)\Lambda\mu + 4(2\psi-1)\mu^2 > 0$. Define the threshold in Proposition A2.3 as

$$\bar{\mu}_c^m = 2\rho\sqrt{\frac{\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{(1-\psi)\Lambda^2+(1+2\psi)\Lambda\mu+4(2\psi-1)\mu^2}}.$$

We show the results for the conventional model and the omnichannel model separately.

Conventional model. Recall fresh-only in-store sales stationary distribution $(1/4, 1/2, 1/4)$. the total-waste rate and optimized taxed profit of fresh-only in-store sales

$$W_t^f = \mu + \frac{3\Lambda}{4}\left(\frac{\mu_c}{\rho+\mu_c}\right)^2, \pi^f(T) = \frac{3\Lambda}{4}\left(V_F - \frac{4}{3}h_s - c\right) - (c+T)\mu.$$

Under full in-store sales, total waste is

$$W_t^\ell = \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu} + \frac{3\Lambda(\Lambda+2\mu)}{3\Lambda+8\mu}\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \frac{2\Lambda\mu}{3\Lambda+8\mu}\frac{\mu_c}{\rho+\mu_c}.$$

Its optimal price and taxed profit are $p_\ell = \min\left\{V_L, \frac{3(\Lambda+2\mu)V_F+2\mu V_L}{3\Lambda+8\mu} - h_s\right\}$ and $\pi^\ell(T) = \Lambda(p_\ell - c) - (c+T)\frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}$. The tax does not enter customer utilities. It therefore does not change the feasible

prices within either regime. Because store waste is constant across prices in a regime, the tax subtracts the same amount at every feasible price. Thus neither of these regime-optimal prices depends on T. Using the customer-waste calculation from Proposition A2.2, the total-waste difference is

$$\begin{aligned} W_t^\ell - W_t^f &= -\frac{\Lambda\mu}{3\Lambda+8\mu} + \frac{\Lambda}{4}\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \frac{2\Lambda\mu}{3\Lambda+8\mu}\frac{\mu_c\rho}{(\rho+\mu_c)^2} \\ &= \frac{\Lambda}{4(3\Lambda+8\mu)(\rho+\mu_c)^2}\left[-4\mu(\rho+\mu_c)^2 + (3\Lambda+8\mu)\mu_c^2 + 8\mu\mu_c\rho\right] \\ &= \frac{\Lambda[(3\Lambda+4\mu)\mu_c^2 - 4\mu\rho^2]}{4(3\Lambda+8\mu)(\rho+\mu_c)^2} \geq 0 \iff \mu_c \geq 2\rho\sqrt{\frac{\mu}{3\Lambda+4\mu}}. \end{aligned}$$

Since $\bar{\mu}_c^m = \frac{\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{(1-\psi)\Lambda^2+(1+2\psi)\Lambda\mu+4(2\psi-1)\mu^2} \geq \frac{\mu}{3\Lambda+4\mu}$, then $\mu_c \geq \bar{\mu}_c^m$ implies this inequality with $W_t^\ell > W_t^f$.

Suppose first that fresh-only sales are optimal without a tax. Then $\pi^f(0) \geq \pi^\ell(0)$. The difference in taxed profits is

$$\pi^\ell(T) - \pi^f(T) = \pi^\ell(0) - \pi^f(0) + T\left[\mu - \frac{2\mu(\Lambda+4\mu)}{3\Lambda+8\mu}\right] = \pi^\ell(0) - \pi^f(0) + \frac{\Lambda\mu}{3\Lambda+8\mu}T.$$

Define

$$T^C = \frac{3\Lambda+8\mu}{\Lambda\mu}[\pi^f(0) - \pi^\ell(0)] \geq 0.$$

For $0 \leq T < T^C$, fresh-only sales remain optimal and $\Delta_{WC} = 0$. At $T = T^C$, the two regimes tie; retaining fresh-only sales follows the conventional tie convention. For $T > T^C$, full in-store sales are optimal and $\Delta_{WC} = W_t^\ell - W_t^f > 0$.

Suppose instead that full in-store sales are optimal without a tax. Then $\pi^\ell(0) \geq \pi^f(0)$. For every $T > 0$, the last profit-difference expression is strictly positive, so full in-store sales remain optimal and $\Delta_{WC} = 0$. If fresh-only sales are infeasible, full in-store sales are the only positive-profit conventional regime, with the same conclusion. Thus $\Delta_{WC} \geq 0$.

When fresh-only sales are feasible and optimal, their price exceeds V_L, so $h_s < 3(V_F - V_L)/4$. Moreover, $\frac{3(\Lambda+2\mu)}{3\Lambda+8\mu} - \frac{3}{4} = \frac{3\Lambda}{4(3\Lambda+8\mu)} > 0$ implies $h_s < \frac{3(\Lambda+2\mu)}{3\Lambda+8\mu}(V_F - V_L)$, and therefore the full-sale candidate price is $p_\ell = V_L$. Substitution into the expression for T^C yields

$$T^C = \frac{(3\Lambda+8\mu)(3V_F - 4V_L - 4h_s) + c(3\Lambda+4\mu)}{4\mu} = \mathcal{T}^m.$$

If the conventional initial regime is full sales, the positive-part definition gives $\mathcal{T}^m = 0$. To justify this also when fresh-only sales are infeasible, note that positive full-sale profit implies $c < p_\ell \leq V_L$, while infeasibility gives $h_s \geq 3(V_F - V_L)/4$. Therefore,

$$\begin{aligned} (3\Lambda+8\mu)(3V_F - 4V_L - 4h_s) + c(3\Lambda+4\mu) &\leq -(3\Lambda+8\mu)V_L + c(3\Lambda+4\mu) \\ &< -(3\Lambda+8\mu)V_L + (3\Lambda+4\mu)V_L = -4\mu V_L < 0. \end{aligned}$$

Thus the positive-part definition gives no positive switching interval in that case.

Omnichannel model. We first identify the feasible candidates that can be optimal. If no all-BOPS outcome is feasible, offering BOPS cannot create a positive-profit BOPS regime at any tax, because utilities and feasibility do not depend on the tax. The omnichannel grocer can choose the conventional configuration, and its optimized outcome coincides with the conventional outcome. Thus $\Delta_{W^B} = \Delta_{W^C} \geq 0$ in this case.

Suppose an all-BOPS outcome is feasible. Lemma A2.2 gives the necessary feasibility condition $h_s - h_b \geq \frac{2(1-\psi)\mu(\Lambda+2\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F - V_L)$. Its optimal price is

$$p_b = \min\left\{ V_F - \frac{2\mu[(1-\psi)\Lambda + (3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}(V_F - V_L) - h_b, \right.$$
$$\left. V_L - \frac{(1-\psi)(\Lambda+2\mu)}{\mu}(V_F - V_L) + \frac{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}{2\mu^2}(h_s - h_b) \right\}.$$

The feasibility condition makes the second price bound at least V_L. For the first price bound, subtract the conventional full-sale participation bound. The difference is

$$\begin{aligned} & h_s - h_b - \left[\frac{2\mu[(1-\psi)\Lambda + (3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} - \frac{2\mu}{3\Lambda+8\mu}\right](V_F - V_L) \\ & \geq \left[\frac{2\mu}{3\Lambda+8\mu} - \frac{2\mu^2}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}\right](V_F - V_L) \\ & = \frac{2(1-\psi)\Lambda\mu(\Lambda+2\mu)(V_F - V_L)}{(3\Lambda+8\mu)\{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2\}} \geq 0. \end{aligned}$$

Thus, both BOPS price bounds are at least their respective conventional full-sale bounds, and $p_b \geq p_\ell$. Since $W_s^b \leq W_s^\ell$, $\pi^b(T) - \pi^\ell(T) = \Lambda(p_b - p_\ell) + (c+T)(W_s^\ell - W_s^b) \geq 0$.

The inequality is strict for $\psi < 1$ under $c > 0$. If $\psi = 1$, then $h_b < h_s$ makes both BOPS price comparisons strict, so the inequality is again strict. Consequently, whenever BOPS is feasible, full in-store sales cannot be optimal in the omnichannel model. Its positive-profit candidates are fresh-only in-store sales and BOPS sales. The BOPS total-waste rate is

$$W_t^b = \frac{2\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} + \Lambda\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \frac{2\Lambda\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}\frac{\mu_c\rho}{(\rho+\mu_c)^2}.$$

To calculate its difference from fresh-only total waste, first note

$$W_s^b - \mu = \frac{2\mu^2(\Lambda+4\mu) - \mu\{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2\}}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} = -\frac{\Lambda\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}.$$

Hence,

$$\begin{aligned} W_t^b - W_t^f &= -\frac{\Lambda\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2} + \frac{\Lambda}{4}\frac{\mu_c^2}{(\rho+\mu_c)^2} + \frac{2\Lambda\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}\frac{\mu_c\rho}{(\rho+\mu_c)^2} \\ &= \frac{\Lambda}{4\{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2\}(\rho+\mu_c)^2} \times \Big\{[(1-\psi)\Lambda^2 + (1+2\psi)\Lambda\mu + 4(2\psi-1)\mu^2]\mu_c^2 \\ &\qquad - 4\mu[(1-\psi)\Lambda+(3-2\psi)\mu]\rho^2\Big\} > 0. \end{aligned}$$

The BOPS taxed profit is $\pi^b(T) = \Lambda(p_b - c) - (c+T)\frac{2\mu^2(\Lambda+4\mu)}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}$. Its optimal price does not depend on T, by the same utility and constant-regime-waste argument used for the conventional model. The profit difference is $\pi^b(T) - \pi^f(T) = \pi^b(0) - \pi^f(0) + \frac{\Lambda\mu[(1-\psi)\Lambda+(3-2\psi)\mu]}{[3\mu+(1-\psi)\Lambda](\Lambda+2\mu)+2\mu^2}T$. The coefficient of T is strictly positive. Thus, if BOPS is already optimal without tax, it remains optimal for every larger tax and $\Delta_{W^B} = 0$. If fresh-only sales are optimal without tax, define

$$T^B = \frac{[3\mu + (1-\psi)\Lambda](\Lambda + 2\mu) + 2\mu^2}{\Lambda\mu[(1-\psi)\Lambda + (3-2\psi)\mu]}[\pi^f(0) - \pi^b(0)] \geq 0.$$

For $0 \leq T < T^B$, fresh-only sales remain optimal and $\Delta_{W^B} = 0$. At equality, the no-BOPS tie convention retains the fresh-only regime. For $T > T^B$, BOPS becomes optimal and $\Delta_{W^B} = W_t^b - W_t^f > 0$. Thus, total waste weakly increases under taxation in the omnichannel model as well.

Comparison across channel configurations. If $h_o < \mathcal{H}_o^m(\psi)$, BOPS is strictly optimal without a tax. It remains optimal under every positive tax by the preceding slope comparison. Its total waste therefore does not change, giving $\Delta_{W^B} = 0 \leq \Delta_{W^C}$.

If $h_o > \mathcal{H}_o^m(\psi)$ and BOPS is infeasible, the two channel configurations have identical feasible operating outcomes and identical optimized waste, so $\Delta_{W^B} = \Delta_{W^C} \geq 0$. If $h_o > \mathcal{H}_o^m(\psi)$ and BOPS is feasible. No BOPS adoption at zero tax means fresh-only sales are optimal in the omnichannel model, because feasible BOPS strictly dominates full in-store sales. The conventional model must also select fresh-only sales at zero tax. Hence $\mathcal{T}^m = T^C$. For every $0 \leq T < \mathcal{T}^m$, the conventional grocer remains fresh-only, so $\Delta_{W^C} = 0$. The omnichannel grocer either remains fresh-only, with $\Delta_{W^B} = 0$, or switches to BOPS $\Delta_{W^B} = W_t^b - W_t^f > 0$. Thus, $\Delta_{W^B} \geq \Delta_{W^C} = 0$ for $0 \leq T < \mathcal{T}^m$. □

S2. Proofs of Results in Section A.3

Throughout this subsection, $\Lambda, \mu, \sigma, h_s, \rho, \mu_c > 0$, $V_F > V_L \geq 0$, and $c \geq 0$. For comparisons involving BOPS, we maintain $0 < \alpha < 1$ and $0 \leq h_o < (1-\alpha)h_s$, so that $h_b = \alpha h_s + h_o < h_s$. Customers participate at zero utility, purchase a less-fresh unit at $p = V_L$, and choose BOPS when its utility weakly exceeds both zero and in-store utility. At a profit tie between feasible fresh-only and full-sale outcomes, the grocer selects fresh-only sales; at a profit tie between offering and not offering BOPS, the grocer does not offer BOPS. In the tax results, we consider only tax levels for which optimal operating profit is positive in both channel configurations.

Proof of Lemma A3.1. An outcome without sales cannot generate positive profit. We consider two regimes with customer participation: customers purchase only fresh products, or they purchase both fresh and less-fresh products.

Fresh-only sales. Suppose $V_L < p \leq V_F$. The purchase rates are $(\lambda_F, \lambda_L) = (\Lambda, 0)$. Substituting these rates into Equation (24) gives $\theta_F = \theta_L = \frac{\sigma}{2\sigma+\Lambda+\mu}, \theta_\emptyset = \frac{\Lambda+\mu}{2\sigma+\Lambda+\mu}$. The customer's expected in-store utility is $u_s = -h_s + \frac{\sigma}{2\sigma+\Lambda+\mu}(V_F - p)$. Then, participation requires $p \leq V_F - (2\sigma + \Lambda + \mu)h_s/\sigma$.

Sales and store waste occur at rates $\Lambda\sigma/(2\sigma+\Lambda+\mu)$ and $\mu\sigma/(2\sigma+\Lambda+\mu)$, respectively. The profit at price p is $\pi(p)=\frac{\sigma[\Lambda(p-c)-c\mu]}{2\sigma+\Lambda+\mu}$, $\frac{d\pi(p)}{dp}=\frac{\Lambda\sigma}{2\sigma+\Lambda+\mu}>0$. Thus, the highest feasible price and its profit are $p_1^C=V_F-\frac{2\sigma+\Lambda+\mu}{\sigma}h_s, \pi_1^C=\frac{\sigma[\Lambda(V_F-c)-c\mu]}{2\sigma+\Lambda+\mu}-\Lambda h_s$. This regime exists if and only if $h_s<\frac{\sigma(V_F-V_L)}{2\sigma+\Lambda+\mu}$.

Full sales. Suppose $0\le p\le V_L$. Both the rates equal Λ, then we have $\theta_F=\frac{\sigma(\Lambda+\mu)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}$, and $\theta_L=\frac{\sigma\mu}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}, \theta_\emptyset=\frac{(\Lambda+\mu)^2}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}$. The participation constraint becomes

$$-h_s+\frac{\sigma[(\Lambda+\mu)V_F+\mu V_L-(\Lambda+2\mu)p]}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}\ge 0.$$

Combining it with $p\le V_L$ gives the optimal price

$$p_2^C=\min\left\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}-\frac{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}{\sigma(\Lambda+2\mu)}h_s\right\}.$$

Indeed, the sales rate is $\Lambda\sigma(\Lambda+2\mu)/[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2]$, the store-waste rate is $\sigma\mu^2/[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2]$, and hence, the profit $\pi(p)=\frac{\sigma[\Lambda(\Lambda+2\mu)(p-c)-c\mu^2]}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}$ is strictly increasing in p. Thus, $\pi_2^C=\pi(p_2^C)$. The price equals V_L precisely when $h_s\le\frac{\sigma(\Lambda+\mu)(V_F-V_L)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}$. A nonnegative price satisfying participation exists precisely when $h_s\le\frac{\sigma[(\Lambda+\mu)V_F+\mu V_L]}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}$.

Comparison of the two regimes. The fresh-state probabilities satisfy

$$\frac{\sigma(\Lambda+\mu)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}-\frac{\sigma}{2\sigma+\Lambda+\mu}=\frac{\Lambda\sigma^2}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2]}>0.$$

It follows that whenever fresh-only sales are feasible, the full-sale price is $p_2^C=V_L$. The profit difference is then

$$\pi_1^C-\pi_2^C=\frac{\sigma[\Lambda(V_F-c)-c\mu]}{2\sigma+\Lambda+\mu}-\Lambda h_s-\frac{\sigma[\Lambda(\Lambda+2\mu)(V_L-c)-c\mu^2]}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}=\Lambda(\hat{h}_s-h_s),$$

where $\hat{h}_s$ is the threshold in the lemma. Therefore, the two conditions for selecting fresh-only sales are its strict feasibility condition and $h_s\le\hat{h}_s$.

To justify the weak inequality used in the statement, consider $h_s=\sigma(V_F-V_L)/(2\sigma+\Lambda+\mu)$. The formal expression for p_1^C is then V_L, which is a full-sale price. Under positive optimal profit, the full-sale optimum at this boundary has positive profit and therefore $V_L>c$. Its sales rate exceeds the formal fresh-only sales rate, since

$$\frac{\Lambda\sigma(\Lambda+2\mu)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}-\frac{\Lambda\sigma}{2\sigma+\Lambda+\mu}=\frac{\Lambda\sigma[\sigma(\Lambda+2\mu)+\mu(\Lambda+\mu)]}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2]}>0.$$

Its store-waste rate is smaller, because

$$\frac{\sigma\mu}{2\sigma+\Lambda+\mu}-\frac{\sigma\mu^2}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}=\frac{\Lambda\mu\sigma(\Lambda+\mu+\sigma)}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2]}>0.$$

The larger sales rate has a strictly positive margin V_L-c, and the smaller store-waste rate weakly reduces procurement cost. Thus $\pi_2^C>\pi_1^C$ at the formal boundary, which implies $\hat{h}_s<\sigma(V_F-V_L)/(2\sigma+\Lambda+\mu)$ there. Consequently, under positive profit, the condition $0<h_s\le\mathcal{H}_s^r$ where $\mathcal{H}_s^r=\min\left\{\frac{\sigma(V_F-V_L)}{2\sigma+\Lambda+\mu}, \hat{h}_s\right\}$ does not mistakenly assign the infeasible boundary to fresh-only sales. The remaining feasible values of h_s select full sales.

Positive-profit conditions. For the two candidate prices, the strict cost bounds for positive profit are $c_1(h_s) = \frac{\Lambda p_1^C}{\Lambda+\mu}, c_2(h_s) = \frac{\Lambda(\Lambda+2\mu)p_2^C}{(\Lambda+\mu)^2}$. The full-sale nonnegative-price bound is necessary for any positive profit. To see this even when considering fresh-only sales, observe that

$$\frac{\sigma[(\Lambda+\mu)V_F+\mu V_L]}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2} - \frac{\sigma V_F}{2\sigma+\Lambda+\mu} = \frac{\sigma[\Lambda\sigma V_F+\mu(2\sigma+\Lambda+\mu)V_L]}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2]} > 0.$$

If the full-sale nonnegative-price bound fails, then $p_2^C < 0$ and $p_1^C < 0$, so neither operating regime is feasible at a nonnegative price.

When fresh-only sales are feasible, positive optimal profit is equivalent to $c < c_1(h_s)$ or $c < c_2(h_s)$, hence to $c < \max\{c_1(h_s), c_2(h_s)\}$. When fresh-only sales are infeasible, the same maximum is valid because $c_1(h_s) \le c_2(h_s)$. In fact, $\frac{(\Lambda+\mu)^2}{\Lambda}[c_2(h_s) - c_1(h_s)] = (\Lambda+2\mu)p_2^C - (\Lambda+\mu)p_1^C$. If $p_2^C = V_L$, infeasibility gives $p_1^C \le V_L$, so the right-hand side is at least $\mu V_L \ge 0$. If the participation constraint determines p_2^C, substitution instead gives

$$(\Lambda+2\mu)p_2^C - (\Lambda+\mu)p_1^C = \mu V_L + \frac{(\Lambda+\mu)(2\sigma+\Lambda+\mu) - \sigma(\Lambda+2\mu) - (\Lambda+\mu)^2}{\sigma} h_s = \mu V_L + \Lambda h_s > 0.$$

The full-sale feasibility bound together with the maximum cost bound is therefore necessary and sufficient. Replacing the displayed fractions by the already defined η and η_0 gives exactly the prices, profits, and positive-profit conditions in the lemma. □

Proof of Lemma A3.2. We first characterize an outcome in which customers use BOPS, and then compare its profit with the two possible conventional outcomes.

Participation and channel preference. When customers use BOPS, both purchase rates equal Λ. The stationary probabilities are therefore the full-sale probabilities derived in the preceding proof. Because a customer pays only when a unit is available, Equation (25) gives

$$u_b = -\alpha h_s - h_o + \frac{\sigma[(\Lambda+\mu)V_F+\mu V_L-(\Lambda+2\mu)p]}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}.$$

The condition $u_b \ge 0$ is equivalent to $p \le p_1$, where

$$p_1 = \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - \frac{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}{\sigma(\Lambda+2\mu)}(\alpha h_s + h_o).$$

For $p > V_L$, an in-store customer rejects a less-fresh unit, and subtracting the two utilities gives

$$u_b - u_s = (1-\alpha)h_s - h_o - \frac{\sigma\mu}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}(p - V_L).$$

Consequently, channel preference requires $p \le p_2$, where

$$p_2 = V_L + \frac{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}{\sigma\mu}[(1-\alpha)h_s - h_o].$$

For $p \le V_L$, an in-store customer accepts either freshness level, so $u_b - u_s = (1-\alpha)h_s - h_o > 0$. In this price range the same bound $p \le p_2$ is automatically satisfied, since $p_2 > V_L$. Thus the two price

bounds characterize BOPS participation and channel preference at all relevant prices. Positive operating profit requires customers to participate; the best BOPS price is $\widehat{p}^B = \min\{p_1, p_2\}$, and $\widehat{\pi}^B = \frac{\sigma[\Lambda(\Lambda+2\mu)(\widehat{p}^B - c) - c\mu^2]}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}$. The profit is strictly increasing in its price. Moreover, p_1, p_2 are continuous and strictly decreasing in h_o, which means that the optimal BOPS profit has the same properties.

Comparison when the conventional optimum has fresh-only sales. Suppose $0 < h_s \le \mathcal{H}_s^r$. The conventional optimum is p_1^C and has profit π_1^C from the preceding proof. At the full-sale price V_L, its advantage over full sales is $\Lambda(\hat{h}_s - h_s)$. Dividing this difference by the full-sale sales rate gives the BOPS price that would match π_1^C:

$$p_3 = V_L + \frac{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}{\sigma(\Lambda+2\mu)}(\hat{h}_s - h_s).$$

Indeed, substituting p_3 into the full-sale profit adds $\Lambda(\hat{h}_s - h_s)$ to the full-sale profit at V_L. Since the full-sale profit is increasing in price, $\widehat{\pi}^B > \pi_1^C \Leftrightarrow \min\{p_1, p_2\} > p_3 \Leftrightarrow p_1 > p_3$ and $p_2 > p_3$. Subtracting p_3 from the first bound and the second bound yields

$$\begin{aligned} p_1 > p_3 &\iff h_o < \frac{\sigma(\Lambda+\mu)(V_F - V_L)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2} - \hat{h}_s + (1-\alpha)h_s = h_{o1}^r, \\ p_2 > p_3 &\iff h_o < (1-\alpha)h_s - \frac{\mu}{\Lambda+2\mu}(\hat{h}_s - h_s) = h_{o2}^r. \end{aligned}$$

Since $\hat{h}_s \ge h_s$ in this conventional regime, $h_{o2}^r \le (1-\alpha)h_s$. Thus, the relevant adoption threshold is $\min\{h_{o1}^r, h_{o2}^r\} \le (1-\alpha)h_s$. At a feasible conventional regime-profit tie, $\hat{h}_s = h_s$, so $h_{o2}^r = (1-\alpha)h_s$. The strict fresh-only feasibility condition and the fresh-probability comparison in the preceding proof give $h_{o1}^r > (1-\alpha)h_s$ there. Hence the adoption threshold equals $(1-\alpha)h_s$ at such a tie.

Comparison when the conventional optimum has full sales. Suppose $h_s > \mathcal{H}_s^r$. Let p_2^C be the conventional full-sale price. The BOPS participation bound exceeds the conventional participation bound by $\frac{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}{\sigma(\Lambda+2\mu)}(h_s - h_b) > 0$, and the BOPS channel-preference bound satisfies $p_2 > V_L$. Since p_2^C is the minimum of V_L and the conventional participation bound, both p_1 and p_2 strictly exceed p_2^C. Consequently $\widehat{p}^B > p_2^C$. The two outcomes have identical purchase rates and stationary probabilities, so $\widehat{\pi}^B - \pi_2^C = \frac{\Lambda\sigma(\Lambda+2\mu)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}(\widehat{p}^B - p_2^C) > 0$. BOPS is therefore strictly profitable for every admissible $h_o < (1-\alpha)h_s$ in this conventional regime. Combining the two regimes defines

$$\mathcal{H}_o^r = \begin{cases} \min\{h_{o1}^r, h_{o2}^r\}, & 0 < h_s \le \mathcal{H}_s^r, \\ (1-\alpha)h_s, & h_s > \mathcal{H}_s^r. \end{cases}$$

This threshold satisfies the inequality stated in the lemma. When $h_o < \mathcal{H}_o^r$, the price $\widehat{p}^B$ is the equilibrium BOPS price, and its profit is strictly greater than the conventional profit.

Conventional fallback and equality. Because offering BOPS is optional, the grocer can always choose the conventional optimum. If $h_o \geq \mathcal{H}_o^r$, the preceding comparisons give $\widehat{\pi}^B \leq \pi^C$, and the stated convention selects no BOPS at equality. Within the maintained strict inequality $h_b < h_s$, this case can occur only when the conventional optimum has fresh-only sales.

For completeness, that in-store outcome also cannot be displaced by a strictly preferable BOPS outcome at its own price when $\widehat{\pi}^B \leq \pi^C$. At the conventional fresh-only optimum $p^C > V_L$, the in-store utility is zero. Suppose customers weakly preferred BOPS there under the fresh-only stationary distribution. Then its utility is nonnegative and $h_s - h_b - \frac{\sigma}{2\sigma+\Lambda+\mu}(p^C - V_L) \geq 0$. Changing to the full-sale stationary distribution increases the fresh-state probability by $\frac{\Lambda\sigma^2}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2]} > 0$ and decreases the less-fresh-state probability by $\frac{\Lambda\sigma(\sigma+\Lambda+\mu)}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2]} > 0$. Because $V_F - p^C > 0$ and $V_L - p^C < 0$, both changes increase $\theta_F(V_F - p^C) + \theta_L(V_L - p^C)$, so BOPS participation still holds. The smaller less-fresh-state probability also makes $h_s - h_b - \theta_L(p^C - V_L)$ strictly larger, so channel preference holds. Hence, BOPS is feasible at p^C. Positive conventional profit implies $p^C > c$; at this price full sales have a larger sales rate and a smaller store-waste rate, as shown in the preceding proof. Their profit therefore strictly exceeds π^C, contradicting $\widehat{\pi}^B \leq \pi^C$. □

Proof of Proposition A3.1. Fix $0 < \mu' < \mu$, keeping all other parameters unchanged. When an argument $x > 0$ is attached to a price, profit, or threshold, the quantity is evaluated with store spoilage rate x. In particular, $\Delta_{\pi^j} = \pi^j_{\mu'} - \pi^j_{\mu}$ for $j \in \{C, B\}$. We first establish how the conventional regime threshold changes, then consider conventional and omnichannel waste and profit separately.

Threshold behavior. Putting the two terms in $\hat{h}_s(x)$ over a common denominator gives $\hat{h}_s(x) = \frac{\sigma}{2\sigma+\Lambda+x}\left[V_F - 2V_L + \frac{(\Lambda+x)(\Lambda V_L+\sigma c)}{\sigma(\Lambda+2x)+(\Lambda+x)^2}\right]$. The derivative of the ratio of $\hat{h}_s(x)$ is decreasing in x whenever $\hat{h}_s(x) > 0$. Moreover, this implies that $\mathcal{H}_s^r(x)$ is continuous and strictly decreasing when it is positive.

Conventional model. Write the store-waste rates in the two sales regimes as $W_s^f(x) = \frac{\sigma x}{2\sigma+\Lambda+x}$, and $W_s^\ell(x) = \frac{\sigma x^2}{\sigma(\Lambda+2x)+(\Lambda+x)^2}$. Both of the store waste rates are increasing in x. Thus, store waste falls whenever the sales regime does not change. At any fixed $x > 0$, full sales generate less store waste than fresh-only sales, since $W_s^f(x) - W_s^\ell(x) = \frac{\Lambda x\sigma(\Lambda+x+\sigma)}{(2\sigma+\Lambda+x)[\sigma(\Lambda+2x)+(\Lambda+x)^2]} > 0$.

If $0 < h_s \leq \mathcal{H}_s^r(\mu)$, the threshold behavior established above gives fresh-only sales at both μ and μ'. Therefore, $W_s^C(\mu') = W_s^f(\mu') < W_s^f(\mu) = W_s^C(\mu)$.

If $h_s > \mathcal{H}_s^r(\mu)$, the initial conventional optimum has full sales. If the reduced-rate optimum also has full sales, the store waste falls. Otherwise, it switches to fresh-only sales. In that case,

$$W_s^C(\mu') \geq W_s^C(\mu) \iff \frac{\mu'}{2\sigma+\Lambda+\mu'} \geq \frac{\mu^2}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}$$
$$\iff \mu'[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2-\mu^2] \geq (2\sigma+\Lambda)\mu^2 \iff \mu' \geq \frac{(2\sigma+\Lambda)\mu^2}{(\sigma+\Lambda)(\Lambda+2\mu)} = \mathcal{M}_{c1}^r.$$

Define $\mathcal{M}_{c2}^r \in (0, \mu)$ satisfying $h_s = \mathcal{H}_s^r(\mathcal{M}_{c2}^r)$. Then, $\mu' \in (\mathcal{M}_{c1}^r, \mathcal{M}_{c2}^r)$ ensures both a switch from full sales to fresh-only sales and a strict increase in conventional store waste.

Omnichannel model. Suppose $0 < h_s \le \mathcal{H}_s^r(\mu)$. Conventional sales are fresh-only at both rates. By Lemma A3.2, BOPS is used at a rate x precisely when $h_o < \mathcal{H}_o^r(x)$. The only case where the store waste increase when spoilage rate descreases to μ' is when BOPS is adopted at μ but not at μ'. Store waste changes from $W_s^\ell(\mu)$ to $W_s^f(\mu')$. Define $\mathcal{H}_{o1}^r = \min\{\mathcal{H}_o^r(\mu), \mathcal{H}_o^r(\mu')\}, \mathcal{H}_{o2}^r = \mathcal{H}_o^r(\mu)$, and $\mathcal{M}_b^r = \frac{(2\sigma+\Lambda)\mu^2}{(\sigma+\Lambda)(\Lambda+2\mu)}$. If $h_o \in (\mathcal{H}_{o1}^r, \mathcal{H}_{o2}^r)$, the grocer adopts BOPS initially and abandons it after the reduction. If also $\mu' \in (\mathcal{M}_b^r, \mu)$, store waste strictly increases. The two optimal-profit comparisons are $\pi_\mu^B > \pi_\mu^C$, and $\pi_{\mu'}^B = \pi_{\mu'}^C$, which implies $\Delta_{\pi^B} - \Delta_{\pi^C} = (\pi_{\mu'}^B - \pi_{\mu'}^C) - (\pi_\mu^B - \pi_\mu^C) = -(\pi_\mu^B - \pi_\mu^C) < 0$.

Suppose $h_s > \mathcal{H}_s^r(\mu)$ and fix a particular $0 < \mu' < \mu$. The conventional initial optimum has full sales, so the maintained $h_o < (1-\alpha)h_s$ implies initial BOPS adoption. At a general rate x, the two BOPS price bounds are $p_1(x) = \frac{(\Lambda+x)V_F + xV_L}{\Lambda+2x} - \frac{\sigma(\Lambda+2x)+(\Lambda+x)^2}{\sigma(\Lambda+2x)}(\alpha h_s + h_o)$ and $p_2(x) = V_L + \frac{\sigma(\Lambda+2x)+(\Lambda+x)^2}{\sigma x}[(1-\alpha)h_s - h_o]$. Subtracting them yields

$$p_1(x) - p_2(x) = \frac{\sigma(\Lambda+2x)+(\Lambda+x)^2}{\sigma x}\frac{\Lambda+x}{\Lambda+2x}[h_o - h_{o3}^r(x)]$$

where $h_{o3}^r(x) = h_s\left(\frac{\Lambda+2x}{\Lambda+x} - \alpha\right) - \frac{\sigma x(V_F - V_L)}{\sigma(\Lambda+2x)+(\Lambda+x)^2}$. Then, $p_2(x) \le p_1(x)$ holds exactly when $h_o \ge h_{o3}^r(x)$.

Define $\mathcal{H}_{o3}^r = \min\{\mathcal{H}_o^r(\mu'), h_{o3}^r(\mu')\}$. If $h_o < \mathcal{H}_{o3}^r$, the grocer continues to adopt BOPS at μ' and the price is $p_1(\mu')$. BOPS has full sales at both rates and $W_s^B(\mu') = W_s^\ell(\mu') < W_s^\ell(\mu) = W_s^B(\mu)$.

Profit gains. In addition to $h_o < \mathcal{H}_{o3}^r$, suppose $\mu' \in (\mathcal{M}_{c1}^r, \mathcal{M}_{c2}^r)$. Define

$$\mathcal{H}_{o4}^r = h_{o3}^r(\mu) + \frac{\mu\sigma}{\Lambda+\mu}\left[\frac{\sigma[\Lambda V_F - c(\Lambda+\mu)] + \mu(2\sigma+\Lambda+\mu)V_L}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2]} - \frac{\sigma[\Lambda V_F - c(\Lambda+\mu')] + \mu'(2\sigma+\Lambda+\mu')V_L}{(2\sigma+\Lambda+\mu')[\sigma(\Lambda+2\mu')+(\Lambda+\mu')^2]}\right]^+.$$

If $h_o \ge \mathcal{H}_{o4}^r$, then $h_o \ge h_{o3}^r(\mu)$, so the initial BOPS price is $p_2(\mu)$; the reduced-rate BOPS price is $p_1(\mu')$ as established above. Then $h_o \ge h_{o3}^r(\mu)$ and $h_o < (1-\alpha)h_s$ imply $0 > (h_{o3}^r(\mu) - (1-\alpha)h_s) = \frac{\mu}{\Lambda+\mu}h_s - \frac{\sigma\mu(V_F-V_L)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}$. Hence, $h_s < \frac{\sigma(\Lambda+\mu)(V_F-V_L)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}$ and the initial conventional full-sale price is V_L. Positive conventional profit implies $V_L > c$. At the formal fresh-only feasibility boundary $h_s = \sigma(V_F - V_L)/(2\sigma+\Lambda+\mu)$, full sales at V_L strictly dominate the formal fresh-only profit, by the sales and waste differences in the proof of Lemma A3.1. Since $\hat{h}_s$ does not depend on h_s, this yields $\hat{h}_s(\mu) < \frac{\sigma(V_F-V_L)}{2\sigma+\Lambda+\mu}$, and $\mathcal{H}_s^r(\mu) = \hat{h}_s(\mu)$. Thus, the assumption $h_s > \mathcal{H}_s^r(\mu)$ gives $h_s - \hat{h}_s(\mu) > 0$. The initial conventional full-sale profit exceeds the formal fresh-only profit by $\Lambda[h_s - \hat{h}_s(\mu)]$.

To perform the profit comparison, write the profit when the BOPS participation bound determines its price as $\pi_1^B(x) = \frac{\sigma\{\Lambda[(\Lambda+x)V_F + xV_L] - c(\Lambda+x)^2\}}{\sigma(\Lambda+2x)+(\Lambda+x)^2} - \Lambda(\alpha h_s + h_o)$, and the fresh-only profit expression is $\pi_1^C(x) = \frac{\sigma[\Lambda(V_F-c)-cx]}{2\sigma+\Lambda+x} - \Lambda h_s$. Subtracting the two profit expressions, we have

$$\pi_1^B(x) - \pi_1^C(x) = \Lambda[(1-\alpha)h_s - h_o] + \frac{\Lambda\sigma\{\sigma[\Lambda V_F - c(\Lambda+x)] + x(2\sigma+\Lambda+x)V_L\}}{(2\sigma+\Lambda+x)[\sigma(\Lambda+2x)+(\Lambda+x)^2]}.$$

At the initial rate μ, the actual BOPS price is $p_2(\mu)$. We have the identities

$$\pi_1^B(\mu) - \pi_\mu^B = \frac{\Lambda(\Lambda+\mu)}{\mu}[h_o - h_{o3}^r(\mu)], \quad \pi_\mu^C - \pi_1^C(\mu) = \Lambda[h_s - \hat{h}_s(\mu)], \quad \pi_{\mu'}^B = \pi_1^B(\mu'), \quad \pi_{\mu'}^C = \pi_1^C(\mu').$$

Substituting these four identities into the difference of profit gains therefore yields

$$\Delta_{\pi^B} - \Delta_{\pi^C} = \Lambda[h_s - \hat{h}_s(\mu)] + \frac{\Lambda(\Lambda+\mu)}{\mu}[h_o - h^r_{o3}(\mu)] + \Lambda\sigma\left[\frac{\sigma[\Lambda V_F - c(\Lambda+\mu')] + \mu'(2\sigma+\Lambda+\mu')V_L}{(2\sigma+\Lambda+\mu')[\sigma(\Lambda+2\mu') + (\Lambda+\mu')^2]} - \frac{\sigma[\Lambda V_F - c(\Lambda+\mu)] + \mu(2\sigma+\Lambda+\mu)V_L}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu) + (\Lambda+\mu)^2]}\right].$$

If the last square bracket is nonnegative, then the second term is nonnegative because $h_o \geq h^r_{o3}(\mu)$. If it is negative, the positive-part term in $\mathcal{H}^r_{o4}$ makes the second term at least its absolute value multiplied by $\Lambda\sigma$. Thus, we have $\Delta_{\pi^B} - \Delta_{\pi^C} \geq \Lambda[h_s - \hat{h}_s(\mu)] > 0$. □

Proof of Proposition A3.2. For each feasible operating outcome, the taxed profit is $\pi_T(p) = (p-c)(\lambda_F\theta_F + \lambda_L\theta_L) - (c+T)\mu\theta_L$. Customer valuations, utilities, purchase decisions at a given price, and stationary probabilities do not directly depend on T. Within either sales regime, the sales rate is positive and independent of price, so profit is strictly increasing in price. The highest price consistent with that regime's participation and channel-preference constraints is consequently independent of the tax. Only the comparison between optimized regime profits changes with T.

Let $\pi^f(0)$ be the optimized fresh-only profit when that regime is feasible. Let $\pi^\ell(0)$ be the optimized conventional full-sale profit, and let $\pi^b(0)$ be the optimized BOPS full-sale profit when BOPS is feasible. The fresh-only and full-sale store-waste rates are $W^f_s = \frac{\sigma\mu}{2\sigma+\Lambda+\mu}$, $W^\ell_s = \frac{\sigma\mu^2}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}$. The corresponding optimized profits at tax T are $\pi^f(T) = \pi^f(0) - TW^f_s$, $\pi^\ell(T) = \pi^\ell(0) - TW^\ell_s$, and $\pi^b(T) = \pi^b(0) - TW^\ell_s$. Notice the difference between the two full-sale profits does not depend on T. The difference between a full-sale profit and the fresh-only profit increases with T, since

$$W^f_s - W^\ell_s = \frac{\sigma\mu\{\sigma(\Lambda+2\mu) + (\Lambda+\mu)^2 - \mu(2\sigma+\Lambda+\mu)\}}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu) + (\Lambda+\mu)^2]} = \frac{\Lambda\mu\sigma(\Lambda+\mu+\sigma)}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu) + (\Lambda+\mu)^2]} > 0.$$

Thus increasing the tax can induce only a switch from fresh-only sales to full sales, never a switch in the opposite direction. At a regime-profit tie, the stipulated fresh-only outcome is selected. If only one sales regime is feasible, the tax cannot change the sales regime.

Effect on store waste. In the conventional model, unchanged sales regimes leave store waste unchanged, while a switch from fresh-only sales to full sales reduces it by the strictly positive difference displayed above. Hence, $W^C_{s,T} \leq W^C_{s,0}$. In the omnichannel model, an in-store full-sale outcome and a BOPS full-sale outcome have the same purchase rates and the same store-waste rate. Switching between these two outcomes does not change waste. The only change in its waste-producing sales regime is from fresh-only sales to full sales, which lowers waste ($W^B_{s,T} \leq W^B_{s,0}$).

Effect on customer waste. The customer's home-spoilage probabilities are unchanged by the restocking delay or the tax. Substituting the stationary probabilities in each regime to Equation (13) gives $W^f_c = \frac{\Lambda\sigma}{2\sigma+\Lambda+\mu}\left(\frac{\mu_c}{\rho+\mu_c}\right)^2$, and $W^\ell_c = \frac{\Lambda\sigma}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}\left[(\Lambda+\mu)\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \mu\frac{\mu_c}{\rho+\mu_c}\right]$. Then,

$$W^\ell_c - W^f_c = \frac{\Lambda\sigma}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu) + (\Lambda+\mu)^2]} \times \left[\Lambda\sigma\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \mu(2\sigma+\Lambda+\mu)\frac{\mu_c}{\rho+\mu_c}\right] > 0.$$

Thus, a fresh-only-to-full-sale switch strictly increases customer waste, whereas an unchanged sales regime leaves it unchanged. Moreover, conventional full sales and BOPS full sales yield the same customer-waste rate because they sell identical freshness mixtures at identical rates. Applying the switching direction separately to the two models gives $W^C_{c,T} \geq W^C_{c,0}$ and $W^B_{c,T} \geq W^B_{c,0}$. □

Proof of Proposition A3.3. We first calculate the total-waste difference between the two sales regimes and characterize the conventional response to the tax. We then characterize the omnichannel response and compare the two models.

Total waste. Adding store and customer waste for two regimes to get the total waste and yields

$$\begin{aligned} W^\ell_t - W^f_t =& \frac{\Lambda\sigma}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2]} \times \left[\Lambda\sigma\left(\frac{\mu_c}{\rho+\mu_c}\right)^2 + \mu(2\sigma+\Lambda+\mu)\frac{\mu_c}{\rho+\mu_c} - \mu(\Lambda+\mu+\sigma)\right] \\ =& \frac{\Lambda\sigma[\sigma(\Lambda+\mu)\mu_c^2 - \mu(\Lambda+\mu)\rho\mu_c - \mu(\Lambda+\mu+\sigma)\rho^2]}{(2\sigma+\Lambda+\mu)[\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2](\rho+\mu_c)^2}. \end{aligned}$$

Define the threshold $\bar{\mu}^r_c$ as the positive root of the numerator:

$$\overline{\mu}^r_c = \frac{\rho}{2}\left[\frac{\mu}{\sigma} + \sqrt{\left(\frac{\mu}{\sigma}\right)^2 + \frac{4\mu(\sigma+\Lambda+\mu)}{\sigma(\Lambda+\mu)}}\right] = \frac{\rho}{2}\left[\frac{\mu}{\sigma} + \sqrt{\left(\frac{\mu}{\sigma}\right)^2 + \frac{4\mu}{\sigma} + \frac{4\mu}{\Lambda+\mu}}\right].$$

Then, $\mu_c \geq \overline{\mu}^r_c$ implies $W^\ell_t - W^f_t \geq 0$ given the numerator is quadratic and only have one root.

Conventional model. When fresh-only sales are feasible, their optimal price is $p_f = V_F - \frac{2\sigma+\Lambda+\mu}{\sigma}h_s$. The conventional full-sale price is $p_\ell = \min\left\{V_L, \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - \frac{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}{\sigma(\Lambda+2\mu)}h_s\right\}$. The feasible price sets are determined by customer participation and acceptance, which do not involve T. At a fixed regime, the sales rate is independent of p and positive, so the tax leaves the maximizing price unchanged. The corresponding candidate profits are

$$\pi^f(T) = \frac{\Lambda\sigma(p_f-c)}{2\sigma+\Lambda+\mu} - (c+T)\frac{\sigma\mu}{2\sigma+\Lambda+\mu} \text{ and } \pi^\ell(T) = \frac{\Lambda\sigma(\Lambda+2\mu)(p_\ell-c)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2} - (c+T)\frac{\sigma\mu^2}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}.$$

If the initial conventional optimum has fresh-only sales, define $T^C = \frac{\pi^f(0)-\pi^\ell(0)}{W^f_s - W^\ell_s} \geq 0$ and

$$\pi^\ell(T) - \pi^f(T) = \pi^\ell(0) - \pi^f(0) + T(W^f_s - W^\ell_s) = (W^f_s - W^\ell_s)(T - T^C).$$

Since $W^f_s - W^\ell_s > 0$, fresh-only sales remain optimal for $0 \leq T \leq T^C$; full sales are strictly optimal for $T > T^C$. Thus, $\Delta_{W^C} = 0$ for $0 \leq T \leq T^C$ and $W^\ell_t - W^f_t$ for $T > T^C$. If full sales are already optimal at zero tax and fresh-only sales are feasible, their initial profit advantage is strictly positive under the tie convention and increases by $T(W^f_s - W^\ell_s)$. Therefore, they remain optimal and $\Delta_{W^C} = 0$. If fresh-only sales are infeasible, only full sales can be an operating optimum, and again $\Delta_{W^C} = 0$. Since $W^\ell_t - W^f_t > 0$, conventional total waste weakly increases under every tax under consideration.

Omnichannel model. The best BOPS full-sale price is

$$p_b = \min\left\{\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - \frac{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}{\sigma(\Lambda+2\mu)}(\alpha h_s + h_o), V_L + \frac{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}{\sigma\mu}[(1-\alpha)h_s - h_o]\right\}.$$

As shown in Lemma A3.2, the first bound strictly exceeds the conventional participation bound because $h_b < h_s$, and the second strictly exceeds V_L. Hence, $p_b > p_\ell$. BOPS price constraints also do not depend on T, so its candidate profit is $\pi^b(T) = \frac{\Lambda\sigma(\Lambda+2\mu)(p_b-c)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2} - (c+T)\frac{\sigma\mu^2}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2}$. Because the two full-sale outcomes have the same store-waste rate, $\pi^b(T) - \pi^\ell(T) = \frac{\Lambda\sigma(\Lambda+2\mu)(p_b-p_\ell)}{\sigma(\Lambda+2\mu)+(\Lambda+\mu)^2} > 0$ for every T. Thus, the conventional full sales can never be selected in the omnichannel model under $h_b < h_s$. Its choice reduces to fresh-only sales and BOPS full sales, with total waste W_t^f and W_t^ℓ.

If fresh-only sales are initially optimal in the omnichannel model, define $T^B = \frac{\pi^f(0)-\pi^b(0)}{W_s^f - W_s^\ell} \geq 0$. The profit difference is $\pi^b(T) - \pi^f(T) = \pi^b(0) - \pi^f(0) + T(W_s^f - W_s^\ell) = (W_s^f - W_s^\ell)(T - T^B)$. No BOPS is selected for $0 \leq T \leq T^B$, including equality; BOPS is strictly optimal for $T > T^B$. Thus, $\Delta_{W^B} = 0$ for $0 \leq T \leq T^B$, and $W_t^\ell - W_t^f$ for $T > T^B$. If BOPS is already optimal without tax, its strict advantage over fresh-only sales increases with T, while its advantage over conventional full sales remains the strictly positive constant just computed. It remains optimal and $\Delta_{W^B} = 0$. If fresh-only sales are infeasible, the same conclusion follows directly from the dominance of BOPS over conventional full sales. These cases prove that omnichannel total waste also weakly increases.

Comparison between the models. We now establish the proposition's two parts.

For part (i), suppose $h_o \geq \mathcal{H}_o^r$, where the threshold is evaluated at zero tax. Lemma A3.2 selects no BOPS at zero tax. Because BOPS strictly dominates conventional full sales under $h_b < h_s$, the initial outcome must be fresh-only in both models. In particular, $\pi^f(0) \geq \pi^b(0) > \pi^\ell(0)$. It follows that both switching thresholds are defined and $T^C - T^B = \frac{\pi^b(0)-\pi^\ell(0)}{W_s^f - W_s^\ell} > 0$ for $0 \leq T^B < T^C$. For $0 \leq T \leq T^B$, both models retain fresh-only sales, so both total-waste changes are zero. For $T^B < T \leq T^C$, the omnichannel grocer uses BOPS and the conventional grocer retains fresh-only sales, so their changes are $W_t^\ell - W_t^f > 0$ and zero, respectively. For $T > T^C$, both models have full sales and both changes equal $W_t^\ell - W_t^f$. At $T = T^B$ or $T = T^C$, the specified no-BOPS and fresh-only conventions give precisely the weak endpoints just stated. Thus $\Delta_{W^B} \geq \Delta_{W^C} \geq 0$ throughout the admitted tax domain. At an interior adoption boundary $h_o = \mathcal{H}_o^r$, the no-tax tie implies $T^B = 0$, which is also covered by these cases.

For part (ii), suppose $h_o < \mathcal{H}_o^r$. BOPS is then strictly optimal at zero tax. Its advantage over conventional full sales stays strictly positive and constant with T, and its advantage over fresh-only sales, when feasible, increases with T. Thus, the omnichannel sales regime remains full sales and $\Delta_{W^B} = 0$. The conventional analysis already established $\Delta_{W^C} \geq 0$. Hence, $\Delta_{W^B} = 0 \leq \Delta_{W^C}$. □

S3. Proofs of Results in Section A.4

Throughout, $\Lambda > 0$, $\mu > 0$, $V_F > V_L \geq 0$, and $0 < \delta < \bar{h}_s$. For the tax results, assume $c \geq 0$, $\rho > 0$, $\mu_c > 0$, and nonnegative tax levels. In the omnichannel model, $0 < \alpha < 1$ and $h_o \geq 0$. The complete uniform distribution function is $G(t) = \frac{t-(\bar{h}_s-\delta)}{2\delta}$ for $h_s - \delta < t < \bar{h}_s + \delta$. For a candidate less-fresh demand rate $x \in [0, \Lambda]$, the stationary probabilities are $\theta_F(x) = \frac{x+\mu}{x+2\mu}$, $\theta_L(x) = \frac{\mu}{x+2\mu}$.

Optimization over feasible customer equilibria. For each channel configuration and tax level, the grocer maximizes profit over all feasible pairs consisting of a price and a customer equilibrium characterized in Lemmas A4.1 and A4.2. The set of feasible pairs does not depend on the tax. If several pairs maximize profit, the grocer selects one with the smallest price. Whenever waste is evaluated at optimal outcomes, we consider tax levels at which such selected operating outcomes exist. A strictly positive supremum of operating profit is attained, and a minimum-price optimizer exists, as proved in the supplementary material. For the total-waste comparison, we maintain the positive-operating-profit restriction of the main tax analysis. This grocer price-tie convention is separate from the customer convention for choosing a channel when channel utilities are equal.

We retain the thresholds definition of $A(p, x)$, $B(p, x)$ and $C(p, x)$ from Section A.4.

Proof of Lemma A4.1. We consider two regimes: customers purchase only fresh products when $V_L < p \leq V_F$, and customers purchase both fresh and less-fresh products when $0 \leq p \leq V_L$.

Fresh-only purchases: $V_L < p \leq V_F$. An in-store customer does not purchase a less-fresh product because $V_L - p < 0$. Therefore, the less-fresh demand rate must be $\lambda_L = 0$. Substituting this demand rate into the stationary probabilities gives $\theta_F = \frac{0+\mu}{0+2\mu} = \frac{1}{2}$, $\theta_L = \frac{\mu}{0+2\mu} = \frac{1}{2}$. The in-store utility of a customer with hassle cost h_s is consequently $u_s(h_s) = -h_s + \frac{1}{2}(V_F - p)$. A customer visits the store if and only if $h_s \leq \frac{V_F-p}{2}$. The fraction of customers who visit is thus $G((V_F - p)/2)$. Every visitor purchases a fresh product when a fresh product is available, and no visitor purchases a less-fresh product. Hence, $\lambda_F = \Lambda G\big(\frac{V_F-p}{2}\big)$, and $\lambda_L = 0$. These rates are uniquely determined by p. They reproduce the stationary probabilities and the participation decision used to derive them, so they form the unique equilibrium in this price region. At $p = V_F$, the participation threshold is zero. Since $\bar{h}_s - \delta > 0$, we have $G(0) = 0$, and therefore $\lambda_F = \lambda_L = 0$.

Purchases of both freshness levels: $0 \leq p \leq V_L$. A customer who visits the store accepts either freshness level. At an equilibrium with less-fresh demand rate x, the fresh demand rate is therefore also x. Substituting the stationary probabilities into in-store utility gives

$$u_s(h_s) = -h_s + \frac{x+\mu}{x+2\mu}(V_F - p) + \frac{\mu}{x+2\mu}(V_L - p) = -h_s + \frac{(x+\mu)V_F + \mu V_L}{x+2\mu} - p = -h_s + A(p, x).$$

Thus, a customer visits if and only if $h_s \leq A(p, x)$, and the visiting fraction is $G(A(p, x))$. The demand rates and the stationary composition are mutually consistent if and only if $x = \Lambda G(A(p, x))$

where $\lambda_F = \lambda_L = x$. We first establish that this equation has a solution. The function $x \to \Lambda G(A(p,x)) - x$ is continuous on $[0,\Lambda]$, because $x + 2\mu > 0$, $A(p,x)$ is continuous in x, and G is continuous. At the two endpoints, $\Lambda G(A(p,0)) \geq 0$, and $\Lambda G(A(p,\Lambda)) - \Lambda \leq 0$. The intermediate value theorem gives at least one solution in $[0,\Lambda]$. We next characterize all such solutions. At $x = 0$, the fixed-point equation holds if and only if $G(A(p,0)) = 0$. By the definition of the uniform distribution function, this is equivalent to $A(p,0) = \frac{V_F+V_L}{2} - p \leq \bar{h}_s - \delta$. At $x = \Lambda$, the equation holds if and only if $G(A(p,\Lambda)) = 1$, which is equivalent to $A(p,\Lambda) = \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu} - p \geq \bar{h}_s + \delta$. For an interior solution $x \in (0,\Lambda)$, the fixed-point equation gives $G(A(p,x)) = \frac{x}{\Lambda} \in (0,1)$. Consequently, $\bar{h}_s - \delta < A(p,x) < \bar{h}_s + \delta$, so the interior formula for G applies and yields:

$$0 = 2\delta x^2 + \left(4\delta\mu + \Lambda[(\bar{h}_s - \delta + p) - V_F]\right) x + \Lambda\mu\left(2(\bar{h}_s - \delta + p) - V_F - V_L\right),$$

Conversely, suppose that $x \in (0,\Lambda)$ satisfies this quadratic equation. Reversing the preceding expansion and dividing by $\Lambda(x+2\mu) > 0$ gives $\frac{2\delta x}{\Lambda} = \frac{(x+\mu)V_F+\mu V_L}{x+2\mu} - p - (\bar{h}_s - \delta)$, which implies $A(p,x) = \bar{h}_s - \delta + \frac{2\delta x}{\Lambda}$. Because $0 < x < \Lambda$, the right-hand side lies strictly between $\bar{h}_s - \delta$ and $\bar{h}_s + \delta$. The interior formula for G is therefore valid, and $\Lambda G(A(p,x)) = \Lambda \frac{A(p,x)-(\bar{h}_s-\delta)}{2\delta} = \Lambda \frac{2\delta x/\Lambda}{2\delta} = x$. Thus, every interior root in $(0,\Lambda)$ is an equilibrium demand rate. Together with the two endpoint tests, this characterizes all equilibria when $0 \leq p \leq V_L$. □

Proof of Lemma A4.2. A customer with in-store hassle cost h_s has BOPS hassle cost $\alpha h_s + h_o$. For a fixed price $p \in [0, V_F]$ and a candidate less-fresh demand rate $x \in [0,\Lambda]$, her utilities are

$$u_s(h_s) = -h_s + \frac{x+\mu}{x+2\mu}(V_F - p) + \frac{\mu}{x+2\mu}(V_L - p)^+ \text{ and } u_b(h_s) = -\alpha h_s - h_o + \frac{(x+\mu)V_F + \mu V_L}{x+2\mu} - p.$$

The participation condition $u_s(h_s) \geq 0$ is equivalent to $h_s \leq A(p,x)$, and $u_b(h_s) \geq 0$ is equivalent to $h_s \leq C(p,x)$. Subtracting the utilities gives $u_s(h_s) - u_b(h_s) = h_o - (1-\alpha)h_s + \frac{\mu}{x+2\mu}(p - V_L)^+$. Since $1 - \alpha > 0$, a customer strictly prefers in-store shopping if and only if $h_s < B(p,x)$ and weakly prefers BOPS if and only if $h_s \geq B(p,x)$. Then, she shops in store when $h_s \leq A(p,x)$ and $h_s < B(p,x)$, and uses BOPS when $B(p,x) \leq h_s \leq C(p,x)$. Thus, the participation fractions are

$$\Phi_s(p,x) = G(\min\{A(p,x), B(p,x)\}) \text{ and } \Phi_b(p,x) = \left[G(C(p,x)) - G(B(p,x))\right]^+.$$

We consider the two price regions separately.

Full sale in store: $0 \leq p \leq V_L$. Every participating customer accepts either freshness level, whether she shops in store or through BOPS. Thus, $\lambda_F = \lambda_L = x$ at equilibrium. In this region, $A(p,x) = \frac{(x+\mu)V_F+\mu V_L}{x+2\mu} - p$, $B(p,x) = \frac{h_o}{1-\alpha}$, $C(p,x) = \frac{A(p,x)-h_o}{\alpha}$. The last two equalities imply $A(p,x) = \alpha C(p,x) + (1-\alpha)B(p,x)$. When $B(p,x) \geq A(p,x)$, we have

$$C(p,x) - A(p,x) = \frac{1-\alpha}{\alpha}\left[A(p,x) - B(p,x)\right] \leq 0 \text{ which implies } C(p,x) \leq A(p,x) \leq B(p,x).$$

It follows that $\Phi_b(p,x)=0$ and $\Phi_s(p,x)=G(A(p,x))$. The equilibrium equation is $x=\Lambda G(A(p,x))$. The endpoint and interior candidate equations are those derived in Lemma A4.1 for $0\le p\le V_L$. Then, $x=0$ requires $\frac{V_F+V_L}{2}-p\le \bar{h}_s-\delta$ and $\frac{h_o}{1-\alpha}\ge \frac{V_F+V_L}{2}-p$; $x=\Lambda$ requires $\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}-p\ge \bar{h}_s+\delta$ and $\frac{h_o}{1-\alpha}\ge \frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}-p$; and an interior candidate $x\in(0,\Lambda)$ must satisfy

$$0=2\delta x^2+\left(4\delta\mu+\Lambda[(\bar{h}_s-\delta+p)-V_F]\right)x+\Lambda\mu\left(2(\bar{h}_s-\delta+p)-V_F-V_L\right)$$

together with $\frac{h_o}{1-\alpha}\ge \frac{(x+\mu)V_F+\mu V_L}{x+2\mu}-p$. Every candidate satisfying these conditions has the in-store participation fraction derived above and zero BOPS participation, so it is also an equilibrium of the omnichannel model. When $B(p,x)<A(p,x)$, the same identity instead gives

$$C(p,x)-A(p,x)=\frac{1-\alpha}{\alpha}\big[A(p,x)-B(p,x)\big]>0 \text{ which implies } B(p,x)<A(p,x)<C(p,x).$$

The participation fractions become $\Phi_s(p,x)=G(B(p,x))$ and $\Phi_b(p,x)=G(C(p,x))-G(B(p,x))$. Then, the total participation fraction is

$$\Phi_s(p,x)+\Phi_b(p,x)=G(B(p,x))+G(C(p,x))-G(B(p,x))=G(C(p,x)).$$

Thus, the equilibrium requires $x=\Lambda G(C(p,x))$. At $x=0$, this equation is equivalent to

$$C(p,0)=\frac{(V_F+V_L)/2-p-h_o}{\alpha}\le \bar{h}_s-\delta \text{ only if } B(p,0)<A(p,0).$$

At $x=\Lambda$, the equation is equivalent to

$$C(p,\Lambda)=\frac{\frac{(\Lambda+\mu)V_F+\mu V_L}{\Lambda+2\mu}-p-h_o}{\alpha}\ge \bar{h}_s+\delta \text{ together with } B(p,\Lambda)<A(p,\Lambda).$$

For an interior equilibrium $x\in(0,\Lambda)$, we have $G(C(p,x))=x/\Lambda\in(0,1)$. Hence, $\bar{h}_s-\delta<C(p,x)<\bar{h}_s+\delta$, and the fixed-point equation becomes

$$0=2\alpha\delta x^2+\left(4\alpha\delta\mu+\Lambda[\alpha(\bar{h}_s-\delta)+h_o+p-V_F]\right)x+\Lambda\mu\left(2\alpha(\bar{h}_s-\delta)+2h_o+2p-V_F-V_L\right),$$

Conversely, suppose that $x\in(0,\Lambda)$ satisfies this quadratic equation and the branch condition $B(p,x)<A(p,x)$. Reversing the expansion and dividing by $\Lambda\alpha(x+2\mu)>0$ gives

$$\frac{2\delta x}{\Lambda}=\frac{\frac{(x+\mu)V_F+\mu V_L}{x+2\mu}-p-h_o}{\alpha}-(\bar{h}_s-\delta)=C(p,x)-(\bar{h}_s-\delta).$$

Therefore, $C(p,x)=\bar{h}_s-\delta+\frac{2\delta x}{\Lambda}\in(\bar{h}_s-\delta,\bar{h}_s+\delta)$. The interior formula for G applies, and $\Lambda G(C(p,x))=\Lambda\frac{C(p,x)-(\bar{h}_s-\delta)}{2\delta}=x$. The branch condition ensures that the channel fractions used above are valid. Hence, the candidate is an equilibrium. This proves both necessity and sufficiency for every endpoint and interior candidate in this branch. Combining the two branches, the total participation fraction is always $G(\max\{A(p,x),C(p,x)\})$. Thus, all equilibria in this price region are precisely the solutions of $x=\Lambda G(\max\{A(p,x),C(p,x)\})$ where $\lambda_F=\lambda_L=x$. To establish existence, we notice $\Lambda G(\max\{A(p,x),C(p,x)\})-x$ is continuous on $[0,\Lambda]$ and its values are nonnegative at $x=0,1$ since G takes values in $[0,1]$. The intermediate value theorem gives at least one fixed point.

Fresh only in store: $V_L < p \le V_F$. In-store customers reject less-fresh products, whereas BOPS customers accept the product regardless of its freshness. Then, only BOPS customers contribute to the less-fresh demand rate, and equilibrium requires $x = \Lambda\Phi_b(p,x) = \Lambda\big[G(C(p,x)) - G(B(p,x))\big]^+$.

In this price region, the thresholds are $A(p,x) = \frac{x+\mu}{x+2\mu}(V_F - p)$, $B(p,x) = \frac{h_o+\mu(p-V_L)/(x+2\mu)}{1-\alpha}$, $C(p,x) = \frac{V_F-p-h_o-\mu(V_F-V_L)/(x+2\mu)}{\alpha}$. These three thresholds are continuous in x. Therefore, the function $\Lambda\big[G(C(p,x)) - G(B(p,x))\big]^+ - x$ is continuous on $[0,\Lambda]$ and has non-negative values at $x = 0, \Lambda$. The intermediate value theorem gives at least one fixed point in $[0,\Lambda]$. For any such fixed point, BOPS customers contribute rate x to the demand for either freshness level, and in-store customers contribute only to fresh demand. Thus, $\lambda_L = x$ and $\lambda_F = x + \Lambda G(\min\{A(p,x), B(p,x)\})$.

These formulas reproduce the participation fractions and the stationary probabilities used to calculate them. Thus, every fixed point, together with the displayed fresh demand rate, gives an equilibrium. Conversely, every equilibrium must satisfy these formulas because all less-fresh purchases arise from BOPS and all fresh purchases arise from the two disjoint groups of participating customers. This establishes the characterization of all equilibrium demand pairs in this price region.

Finally, at $p = V_F$ we have $A(V_F,x) = 0$ and $C(V_F,x) = -\frac{h_o+\mu(V_F-V_L)/(x+2\mu)}{\alpha} < 0$ for all $x \in [0,\Lambda]$. Since $\bar{h}_s - \delta > 0$, it follows that $G(C(V_F,x)) = 0$. The fixed-point equation therefore has only the solution $x = 0$. Moreover, $G(\min\{A(V_F,0), B(V_F,0)\}) = G(0) = 0$, and $\lambda_F = \lambda_L = 0$ is the unique equilibrium at $p = V_F$. Combining the two price regions completes the proof of the equilibrium characterization and existence. □

Existence of the selected optimum. Fix a model $j \in \{C,B\}$ and a tax $T \ge 0$, and suppose that the supremum π_T^j of operating profit over all feasible price–equilibrium pairs is strictly positive. The fresh demand functions are $\lambda_F^C(p,x) = \Lambda G(A(p,x))$ and $\lambda_F^B(p,x) = \Lambda G(\max\{A(p,x), C(p,x)\})$, where the second expression follows from $A(p,x) = \alpha C(p,x) + (1-\alpha)B(p,x)$. The definitions of A and C imply that both functions are continuous in (p,x) and nondecreasing in x. Hence, $\Pi_T^j(p,x) = (p-c)[(x+\mu)\lambda_F^j(p,x) + \mu x]/(x+2\mu) - (c+T)\mu^2/(x+2\mu)$ is continuous on $[0,V_F] \times [0,\Lambda]$. Choose feasible pairs (p_n, x_n) whose profits converge to π_T^j, and extract a subsequence converging to (p_*, x_*). Continuity gives $\Pi_T^j(p_*, x_*) = \pi_T^j$. After restricting to one price region, the equilibrium equations pass to the limit unless $p_n > V_L$ and $p_* = V_L$. In this remaining case, $\lambda_F^j(V_L, x_*) \ge x_*$, because fresh demand is at least less-fresh demand along the sequence. At $p = V_L$, equilibrium requires $x = \lambda_F^j(V_L, x)$. Since $\lambda_F^j(V_L, x_*) \ge x_*$ and $\lambda_F^j(V_L, \Lambda) \le \Lambda$, continuity gives a fixed point $\widehat{x} \in [x_*, \Lambda]$; monotonicity then implies $\widehat{x} = \lambda_F^j(V_L, \widehat{x}) \ge \lambda_F^j(V_L, x_*)$. The feasible outcome $(V_L, \widehat{x})$ has sales rate $\widehat{x}$, whereas the limiting sales rate satisfies $\big[(x_*+\mu)\lambda_F^j(V_L,x_*) + \mu x_*\big]/(x_*+2\mu) \le \lambda_F^j(V_L,x_*) \le \widehat{x}$. Its store-waste rate is also weakly lower because $\widehat{x} \ge x_*$. Moreover, $\Pi_T^j(V_L, x_*) = \pi_T^j > 0$ implies $V_L > c$. Therefore, the higher sales and lower store waste give $\Pi_T^j(V_L, \widehat{x}) \ge \pi_T^j$, with equality by the definition of the supremum. Thus, the positive profit supremum is attained in every case.

Let $p_{\min}$ be the infimum of the prices among profit-maximizing feasible outcomes, and choose a sequence of such outcomes whose prices converge to $p_{\min}$. Their profits all equal $\pi_T^j > 0$. Extract a convergent subsequence of their less-fresh demand rates and apply the preceding argument. If the limiting pair is feasible, it is an optimizer at price $p_{\min}$. The only possible failure of feasibility occurs when prices approach V_L from above, in which case $p_{\min} = V_L$, and the preceding construction supplies a feasible optimizer at exactly that price. Hence, a minimum-price profit-maximizing outcome exists, and the stated price-selection convention is well-defined. □

Proof of Proposition A4.1. The argument applies to the conventional and omnichannel models. We omit the superscripts denoting the channel configuration except when their customer-equilibrium equations differ. Fix two tax levels satisfying $0 \le T_1 < T_2$. At tax level T_i, let p_{T_i}, λ_{F,T_i}, and λ_{L,T_i} denote the price and the equilibrium purchase rates at the selected optimal outcome, for $i = 1, 2$. For an equilibrium with price p and purchase rates λ_F, λ_L, the stationary probabilities are $\theta_F = \frac{\lambda_L+\mu}{\lambda_L+2\mu}$ and $\theta_L = \frac{\mu}{\lambda_L+2\mu}$. The taxed profit is $\pi_T(p, \lambda_L) = (p-c)\frac{(\lambda_L+\mu)\lambda_F+\mu\lambda_L}{\lambda_L+2\mu} - (c+T)\frac{\mu^2}{\lambda_L+2\mu}$, where λ_F is the fresh purchase rate associated. At a fixed equilibrium outcome, the tax enters profit only through store waste. Hence, $\pi_{T_2}(p, \lambda_L) = \pi_{T_1}(p, \lambda_L) - (T_2 - T_1)\frac{\mu^2}{\lambda_L+2\mu}$. Customer utilities do not depend on the tax, so an equilibrium feasible at either tax level is also feasible at the other tax level. We discuss the effects on store waste and customer waste separately.

Effect on store waste. If the optimality is selected at T_1, $\pi_{T_1}(p_{T_1}, \lambda_{L,T_1}) \ge \pi_{T_1}(p_{T_2}, \lambda_{L,T_2})$. Otherwise, $\pi_{T_1}(p_{T_2}, \lambda_{L,T_2}) - (T_2 - T_1)\frac{\mu^2}{\lambda_{L,T_2}+2\mu} \ge \pi_{T_1}(p_{T_1}, \lambda_{L,T_1}) - (T_2 - T_1)\frac{\mu^2}{\lambda_{L,T_1}+2\mu}$. Adding these inequalities yields $(T_2 - T_1)\left[\frac{\mu^2}{\lambda_{L,T_1}+2\mu} - \frac{\mu^2}{\lambda_{L,T_2}+2\mu}\right] \ge 0$. Then, $W_{s,T_2} = \frac{\mu^2}{\lambda_{L,T_2}+2\mu} \le \frac{\mu^2}{\lambda_{L,T_1}+2\mu} = W_{s,T_1}$ since $T_2 > T_1$. Because $\mu^2/(x+2\mu)$ is strictly decreasing in x, we also obtain $\lambda_{L,T_2} \ge \lambda_{L,T_1}$.

Effect on customer waste We first establish that the actual total sales rate is weakly higher at T_2. To do so, we need the dependence of customer demand on price and on a candidate less-fresh purchase rate x. In the conventional model, the fresh purchase rate associated with a candidate pair (p, x) is $\Lambda G(A(p, x))$. In the omnichannel model, the corresponding expression is $\Lambda\left[G(\min\{A(p,x), B(p,x)\}) + \left(G(C(p,x)) - G(B(p,x))\right)^+\right]$. The threshold identity holds throughout $p \in [0, V_F]$, not only below V_L. Also, notice that $\alpha C(p,x) + (1-\alpha)B(p,x) = A(p,x)$. If $B(p,x) \ge A(p,x)$, then this identity implies $C(p,x) \le A(p,x) \le B(p,x)$; the omnichannel fresh purchase rate is therefore $\Lambda G(A(p,x))$. If $B(p,x) < A(p,x)$, the identity implies $B(p,x) < A(p,x) < C(p,x)$; the omnichannel fresh purchase rate is then $\Lambda G(C(p,x))$. Thus, at every candidate pair, the omnichannel fresh purchase rate equals $\Lambda G(\max\{A(p,x), C(p,x)\})$. At a fixed candidate rate x, both $A(p,x)$ and $C(p,x)$ are nonincreasing in p. At a fixed price p, they are nondecreasing in x. Since G is nondecreasing, the fresh purchase rate in either model is nonincreasing in price at fixed x and nondecreasing in x at fixed price. The actual total sales rate at a candidate pair is obtained by substituting its fresh purchase rate in $\frac{(x+\mu)\lambda_F+\mu x}{x+2\mu}$. At fixed x, this expression is nonincreasing in

price. At fixed price, it is strictly increasing in x and the actual fresh purchase rate also weakly increases with x. Suppose, to obtain a contradiction, that

$$\frac{(\lambda_{L,T_2}+\mu)\lambda_{F,T_2}+\mu\lambda_{L,T_2}}{\lambda_{L,T_2}+2\mu} < \frac{(\lambda_{L,T_1}+\mu)\lambda_{F,T_1}+\mu\lambda_{L,T_1}}{\lambda_{L,T_1}+2\mu}. \tag{S.1}$$

We have already shown that $\lambda_{L,T_2} \geq \lambda_{L,T_1}$. The preceding sales comparisons imply $p_{T_2} > p_{T_1}$. If $\lambda_{L,T_2} = \lambda_{L,T_1}$, store waste is the same at the two outcomes. The two optimality inequalities then imply that the first outcome is also optimal at T_2. Its smaller price contradicts the smallest-price selection rule. Thus, $\lambda_{L,T_2} > \lambda_{L,T_1}$. In particular, $\lambda_{L,T_2} > 0$, so the actual total sales rate at T_2 is strictly positive. We consider the two possible relations between p_{T_1} and c.

Case 1: $p_{T_1} \leq c$. Evaluating both outcomes at tax level T_1, their profit difference is

$$\pi_{T_1}(p_{T_2},\lambda_{L,T_2}) - \pi_{T_1}(p_{T_1},\lambda_{L,T_1}) = (p_{T_2}-p_{T_1})\frac{(\lambda_{L,T_2}+\mu)\lambda_{F,T_2}+\mu\lambda_{L,T_2}}{\lambda_{L,T_2}+2\mu} + (c+T_1)\left[\frac{\mu^2}{\lambda_{L,T_1}+2\mu} - \frac{\mu^2}{\lambda_{L,T_2}+2\mu}\right]$$
$$+ (p_{T_1}-c)\left[\frac{(\lambda_{L,T_2}+\mu)\lambda_{F,T_2}+\mu\lambda_{L,T_2}}{\lambda_{L,T_2}+2\mu} - \frac{(\lambda_{L,T_1}+\mu)\lambda_{F,T_1}+\mu\lambda_{L,T_1}}{\lambda_{L,T_1}+2\mu}\right] > 0.$$

The first term is strictly positive because $p_{T_2} > p_{T_1}$ and sales at the second outcome are positive. The second term is nonnegative by $p_{T_1} \leq c$ and Equation (S.1). The third term is nonnegative because $c+T_1 \geq 0$ and store waste is lower at the second outcome. This strict profit improvement contradicts optimality at T_1.

Case 2: $p_{T_1} > c$. We show that the first price admits a feasible customer equilibrium with a less-fresh purchase rate at least λ_{L,T_2}. In the conventional model, the right-hand side of the less-fresh demand equation is $\Lambda G(A(p,x))$ when $p \leq V_L$ and is zero when $p > V_L$. For fixed x, it is nonincreasing in p, including the downward jump that may occur when the price rises above V_L. In the omnichannel model, the right-hand side is $\Lambda G(\max\{A(p,x), C(p,x)\})$ when $p \leq V_L$. When $p > V_L$, it equals $\Lambda\big[G(C(p,x)) - G(B(p,x))\big]^+$. In the latter price range, $C(p,x)$ decreases with p, whereas $B(p,x)$ increases with p. This right-hand side is nonincreasing in price. At $p = V_L$, the same BOPS contribution is present as in the limit from above, and the in-store contribution is nonnegative. Thus, raising the price across V_L cannot increase the right-hand side. The required price comparison holds in both models over the entire price interval.

Since $p_{T_1} < p_{T_2}$, evaluating the appropriate right-hand side at $(p_{T_1}, \lambda_{L,T_2})$ gives a value at least its value at $(p_{T_2}, \lambda_{L,T_2})$, which equals λ_{L,T_2}. At the candidate rate $x = \Lambda$, the right-hand side is at most Λ. For fixed p_{T_1}, it is continuous in x, since the thresholds and G are continuous and the positive-part, minimum, and maximum operations preserve continuity. The intermediate value theorem therefore gives a fixed point $\widehat{\lambda}_L \in [\lambda_{L,T_2}, \Lambda]$ at the price p_{T_1}. Let $\widehat{\lambda}_F$ be the corresponding fresh purchase rate. This is a feasible equilibrium at T_1. Since $\widehat{\lambda}_L \geq \lambda_{L,T_2} > \lambda_{L,T_1}$, the strict

increase of actual total sales with the candidate less-fresh rate at a fixed price gives $\frac{(\widehat{\lambda}_L+\mu)\widehat{\lambda}_F+\mu\widehat{\lambda}_L}{\widehat{\lambda}_L+2\mu} > \frac{(\lambda_{L,T_1}+\mu)\lambda_{F,T_1}+\mu\lambda_{L,T_1}}{\lambda_{L,T_1}+2\mu}$ and $\frac{\mu^2}{\widehat{\lambda}_L+2\mu} < \frac{\mu^2}{\lambda_{L,T_1}+2\mu}$. Therefore,

$$\begin{aligned}\pi_{T_1}(p_{T_1},\widehat{\lambda}_L)-\pi_{T_1}(p_{T_1},\lambda_{L,T_1}) =& (p_{T_1}-c)\left[\frac{(\widehat{\lambda}_L+\mu)\widehat{\lambda}_F+\mu\widehat{\lambda}_L}{\widehat{\lambda}_L+2\mu}-\frac{(\lambda_{L,T_1}+\mu)\lambda_{F,T_1}+\mu\lambda_{L,T_1}}{\lambda_{L,T_1}+2\mu}\right]\\ &+(c+T_1)\left[\frac{\mu^2}{\lambda_{L,T_1}+2\mu}-\frac{\mu^2}{\widehat{\lambda}_L+2\mu}\right]>0.\end{aligned}$$

This again contradicts optimality at T_1.

Both cases lead to contradictions. Then, $\frac{(\lambda_{L,T_2}+\mu)\lambda_{F,T_2}+\mu\lambda_{L,T_2}}{\lambda_{L,T_2}+2\mu} \geq \frac{(\lambda_{L,T_1}+\mu)\lambda_{F,T_1}+\mu\lambda_{L,T_1}}{\lambda_{L,T_1}+2\mu}$ and e can now compare customer waste. At either selected equilibrium,

$$\begin{aligned}W_{c,T_i} &= \left(\frac{\mu_c}{\rho+\mu_c}\right)^2\frac{(\lambda_{L,T_i}+\mu)\lambda_{F,T_i}}{\lambda_{L,T_i}+2\mu}+\frac{\mu_c}{\rho+\mu_c}\frac{\mu\lambda_{L,T_i}}{\lambda_{L,T_i}+2\mu}\\ &= \left(\frac{\mu_c}{\rho+\mu_c}\right)^2\frac{(\lambda_{L,T_i}+\mu)\lambda_{F,T_i}+\mu\lambda_{L,T_i}}{\lambda_{L,T_i}+2\mu}+\frac{\rho\mu_c}{(\rho+\mu_c)^2}\frac{\mu\lambda_{L,T_i}}{\lambda_{L,T_i}+2\mu}.\end{aligned}$$

The first term is weakly higher at T_2 because actual total sales are weakly higher. The second term is also weakly higher, because $\lambda_{L,T_2} \geq \lambda_{L,T_1}$ and $\frac{d}{dx}\frac{\mu x}{x+2\mu} = \frac{2\mu^2}{(x+2\mu)^2} > 0$. It follows that $W_{c,T_2} \geq W_{c,T_1}$. This applies to both channel configurations. Taking $T_1 = 0$ and $T_2 = T > 0$ completes the proof. □

Proof of Proposition A4.2. Throughout the proof, assume $\mu_c > \rho$. We examine the conventional and omnichannel models separately. For each model, let p_0 and p_T denote the selected optimal prices before and after the tax, and let $\lambda_{F,0}, \lambda_{L,0}$ and $\lambda_{F,T}, \lambda_{L,T}$ denote the associated fresh- and less-fresh purchase rates. We omit the model superscript within each part of the proof.

Conventional model By the store-waste comparison in Proposition A4.1, $\frac{\mu^2}{\lambda_{L,T}+2\mu} \leq \frac{\mu^2}{\lambda_{L,0}+2\mu}$. Since $\mu > 0$, this implies $\lambda_{L,T} \geq \lambda_{L,0}$. We first show that $p_T \leq p_0$. Strictly positive optimized profit at tax zero gives $(p_0 - c)\left[\frac{\lambda_{L,0}+\mu}{\lambda_{L,0}+2\mu}\lambda_{F,0} + \frac{\mu\lambda_{L,0}}{\lambda_{L,0}+2\mu}\right] > \frac{c\mu^2}{\lambda_{L,0}+2\mu}$, which implies $p_0 > c$. Suppose, toward a contradiction, that $p_T > p_0$. If $\lambda_{L,T} = \lambda_{L,0}$, both selected outcomes have the same store-waste rate. Their profits are therefore reduced by the same amount when the tax increases from zero to T. Optimality at the two tax levels then implies that their untaxed profits are equal. Consequently, the outcome selected before taxation is also optimal at tax T. Its price is smaller than p_T, contrary to the convention of selecting the smallest optimal price.

It remains to consider $\lambda_{L,T} > \lambda_{L,0}$. Then $\lambda_{L,T} > 0$, so Lemma A4.1 implies $p_T \leq V_L$. Since $p_0 < p_T$, we also have $p_0 \leq V_L$. For a candidate less-fresh purchase rate $x \in [0, \Lambda]$, $\Lambda G(A(p_0, x))$ is continuous. Since $A(p, x)$ is nonincreasing in p, $\Lambda G(A(p_0, \lambda_{L,T})) \geq \Lambda G(A(p_T, \lambda_{L,T})) = \lambda_{L,T}$. With $\Lambda G(A(p_0, \Lambda)) \leq \Lambda$, the intermediate value theorem applied to $\Lambda G(A(p_0, x)) - x$ on $[\lambda_{L,T}, \Lambda]$ gives an equilibrium $\widehat{\lambda}_L \geq \lambda_{L,T}$ at price p_0. In this price region, fresh- and less-fresh purchase rates coincide,

so actual total sales equal the purchase rate. The untaxed profit difference between this alternative equilibrium and the selected baseline outcome is

$$\left[(p_0-c)\widehat{\lambda}_L-\frac{c\mu^2}{\widehat{\lambda}_L+2\mu}\right]-\left[(p_0-c)\lambda_{L,0}-\frac{c\mu^2}{\lambda_{L,0}+2\mu}\right]=(p_0-c)(\widehat{\lambda}_L-\lambda_{L,0})+c\mu^2\left[\frac{1}{\lambda_{L,0}+2\mu}-\frac{1}{\widehat{\lambda}_L+2\mu}\right]>0.$$

This contradicts optimality of the baseline outcome. Hence $p_T \le p_0$.

At every conventional equilibrium, $\lambda_F = \Lambda G(A(p,\lambda_L))$. The threshold $A(p,x)$ is nonincreasing in price and nondecreasing in x. Using $p_T \le p_0$ and $\lambda_{L,T} \ge \lambda_{L,0}$, we obtain

$$\lambda_{F,T} = \Lambda G(A(p_T,\lambda_{L,T})) \ge \Lambda G(A(p_0,\lambda_{L,0})) = \lambda_{F,0}.$$

The actual rate of fresh products sold therefore also weakly increases. More explicitly,

$$\frac{\lambda_{L,T}+\mu}{\lambda_{L,T}+2\mu}\lambda_{F,T}-\frac{\lambda_{L,0}+\mu}{\lambda_{L,0}+2\mu}\lambda_{F,0}=\frac{\lambda_{L,T}+\mu}{\lambda_{L,T}+2\mu}(\lambda_{F,T}-\lambda_{F,0})+\frac{\mu\lambda_{F,0}(\lambda_{L,T}-\lambda_{L,0})}{(\lambda_{L,T}+2\mu)(\lambda_{L,0}+2\mu)}\ge 0.$$

At a tax level $\tau \in \{0,T\}$, total waste is

$$W^C_{t,\tau}=\frac{\mu^2}{\lambda_{L,\tau}+2\mu}+\left(\frac{\mu_c}{\rho+\mu_c}\right)^2\frac{\lambda_{L,\tau}+\mu}{\lambda_{L,\tau}+2\mu}\lambda_{F,\tau}+\frac{\mu_c}{\rho+\mu_c}\frac{\mu\lambda_{L,\tau}}{\lambda_{L,\tau}+2\mu}.$$

Since $\frac{\mu\lambda_{L,\tau}}{\lambda_{L,\tau}+2\mu}=\mu-\frac{2\mu^2}{\lambda_{L,\tau}+2\mu}$, we may rewrite this expression as

$$W^C_{t,\tau}=\left(\frac{\mu_c}{\rho+\mu_c}\right)^2\frac{\lambda_{L,\tau}+\mu}{\lambda_{L,\tau}+2\mu}\lambda_{F,\tau}+\frac{\mu\mu_c}{\rho+\mu_c}+\frac{\rho-\mu_c}{\rho+\mu_c}\frac{\mu^2}{\lambda_{L,\tau}+2\mu}.$$

Subtracting the value at tax zero from the value at tax T yields

$$W^C_{t,T}-W^C_{t,0}=\left(\frac{\mu_c}{\rho+\mu_c}\right)^2\left[\frac{\lambda_{L,T}+\mu}{\lambda_{L,T}+2\mu}\lambda_{F,T}-\frac{\lambda_{L,0}+\mu}{\lambda_{L,0}+2\mu}\lambda_{F,0}\right]+\frac{\mu_c-\rho}{\rho+\mu_c}\left[\frac{\mu^2}{\lambda_{L,0}+2\mu}-\frac{\mu^2}{\lambda_{L,T}+2\mu}\right]\ge 0.$$

The first bracket is nonnegative because actual fresh sales weakly increase. The second bracket is nonnegative because store waste weakly decreases. $\mu_c > \rho$ makes the second coefficient positive.

Omnichannel model Proposition A4.1 again gives $\lambda_{L,T} \ge \lambda_{L,0}$. We show that the optimal price and fresh purchase rate satisfy the same comparisons as in the conventional model, while allowing less-fresh BOPS purchases when $p > V_L$. The identity $A(p,x) = \alpha C(p,x) + (1-\alpha)B(p,x)$ implies that the fresh purchase rate at an omnichannel equilibrium is $\lambda_F = \Lambda G(\max\{A(p,\lambda_L), C(p,\lambda_L)\})$. Indeed, if $B(p,x) \ge A(p,x)$, then $C(p,x) \le A(p,x) \le B(p,x)$, no customers use BOPS, and the fresh purchase rate is $\Lambda G(A(p,x))$. If $B(p,x) < A(p,x)$, then $B(p,x) < A(p,x) < C(p,x)$, and summing the in-store rate $\Lambda G(B(p,x))$ and BOPS rate $\Lambda[G(C(p,x)) - G(B(p,x))]$ gives $\Lambda G(C(p,x))$. These two cases establish the displayed expression, including thresholds outside the support of G.

For a candidate $x \in [0,\Lambda]$, the complete less-fresh purchase map is

$$\begin{cases} \Lambda G(\max\{A(p,x), C(p,x)\}), & 0 \le p \le V_L, \\ \Lambda\big[G(C(p,x)) - G(B(p,x))\big]^+, & V_L < p \le V_F. \end{cases}$$

At a fixed price this map is continuous in x and takes values in $[0,\Lambda]$. At a fixed x, it is nonincreasing in price: below V_L, both $A(p,x)$ and $C(p,x)$ are nonincreasing in price; above V_L, $C(p,x)$ decreases while $B(p,x)$ increases; when price crosses V_L upward, the nonnegative in-store contribution to less-fresh purchases disappears. Thus, the crossing cannot create an increase in the map. Strictly positive untaxed profit means that $(p_0-c)\frac{(\lambda_{L,0}+\mu)\lambda_{F,0}+\mu\lambda_{L,0}}{\lambda_{L,0}+2\mu}-\frac{c\mu^2}{\lambda_{L,0}+2\mu}>0$, which implies $p_0>c$.

Suppose, toward a contradiction, that $p_T>p_0$. If $\lambda_{L,T}=\lambda_{L,0}$, the two outcomes have identical store waste. Let π_0 denotes untaxed profit at the indicated feasible equilibrium. Optimality before and after taxation gives $\pi_0(p_0,\lambda_{L,0})\geq\pi_0(p_T,\lambda_{L,T})$ and $\pi_0(p_T,\lambda_{L,T})-\frac{T\mu^2}{\lambda_{L,T}+2\mu}\geq\pi_0(p_0,\lambda_{L,0})-\frac{T\mu^2}{\lambda_{L,0}+2\mu}$. Because the tax deductions are identical, the two untaxed profits are equal. The baseline outcome therefore also maximizes profit at tax T. Its strictly smaller price contradicts the minimum-price convention. If $\lambda_{L,T}>\lambda_{L,0}$, evaluate the complete less-fresh purchase map at price p_0. Its value at $x=\lambda_{L,T}$ is at least its value at price p_T, namely $\lambda_{L,T}$, and its value at $x=\Lambda$ is at most Λ. Continuity and the intermediate value theorem give a feasible customer equilibrium at price p_0 with $\widehat{\lambda}_L\geq\lambda_{L,T}>\lambda_{L,0}$. Both thresholds governing fresh purchases weakly increase with the candidate less-fresh purchase rate. For $A(p,x)$, this follows from its derivative displayed in the conventional-model part. For $C(p,x)$, $\frac{\partial C(p,x)}{\partial x}=\frac{\mu(V_F-V_L)}{\alpha(x+2\mu)^2}>0$. It follows that the fresh purchase rate at the alternative equilibrium is at least $\lambda_{F,0}$. Thus, the increase in its actual sales relative to the baseline outcome is at least $\left[\frac{\widehat{\lambda}_L+\mu}{\widehat{\lambda}_L+2\mu}\lambda_{F,0}+\frac{\mu\widehat{\lambda}_L}{\widehat{\lambda}_L+2\mu}\right]-\left[\frac{\lambda_{L,0}+\mu}{\lambda_{L,0}+2\mu}\lambda_{F,0}+\frac{\mu\lambda_{L,0}}{\lambda_{L,0}+2\mu}\right]=\frac{\mu(\lambda_{F,0}+2\mu)(\widehat{\lambda}_L-\lambda_{L,0})}{(\widehat{\lambda}_L+2\mu)(\lambda_{L,0}+2\mu)}>0$. The alternative equilibrium also has strictly lower store waste. Let $\widehat{\lambda}_F$ be its fresh purchase rate. Its untaxed profit difference from the baseline outcome is

$$\pi_0(p_0,\widehat{\lambda}_L)-\pi_0(p_0,\lambda_{L,0})=(p_0-c)\left[\frac{(\widehat{\lambda}_L+\mu)\widehat{\lambda}_F+\mu\widehat{\lambda}_L}{\widehat{\lambda}_L+2\mu}-\frac{(\lambda_{L,0}+\mu)\lambda_{F,0}+\mu\lambda_{L,0}}{\lambda_{L,0}+2\mu}\right]+c\left[\frac{\mu^2}{\lambda_{L,0}+2\mu}-\frac{\mu^2}{\widehat{\lambda}_L+2\mu}\right]>0.$$

This contradicts baseline optimality. We conclude that $p_T\leq p_0$.

Both $A(p,x)$ and $C(p,x)$ are nonincreasing in price and nondecreasing in x. Hence,

$$\lambda_{F,T}=\Lambda G(\max\{A(p_T,\lambda_{L,T}),C(p_T,\lambda_{L,T})\})\geq\Lambda G(\max\{A(p_0,\lambda_{L,0}),C(p_0,\lambda_{L,0})\})=\lambda_{F,0}.$$

Since $\lambda_{L,T}\geq\lambda_{L,0}$, we have $\frac{\lambda_{L,T}+\mu}{\lambda_{L,T}+2\mu}\lambda_{F,T}-\frac{\lambda_{L,0}+\mu}{\lambda_{L,0}+2\mu}\lambda_{F,0}=\frac{\lambda_{L,T}+\mu}{\lambda_{L,T}+2\mu}(\lambda_{F,T}-\lambda_{F,0})+\frac{\mu\lambda_{F,0}(\lambda_{L,T}-\lambda_{L,0})}{(\lambda_{L,T}+2\mu)(\lambda_{L,0}+2\mu)}\geq 0$.

At a tax level $\tau\in\{0,T\}$, the omnichannel total-waste rate is the sum of its store waste, household waste from fresh sales, and household waste from less-fresh sales:

$$\begin{aligned}W^B_{t,\tau}&=\frac{\mu^2}{\lambda_{L,\tau}+2\mu}+\left(\frac{\mu_c}{\rho+\mu_c}\right)^2\frac{\lambda_{L,\tau}+\mu}{\lambda_{L,\tau}+2\mu}\lambda_{F,\tau}+\frac{\mu_c}{\rho+\mu_c}\frac{\mu\lambda_{L,\tau}}{\lambda_{L,\tau}+2\mu}\\&=\left(\frac{\mu_c}{\rho+\mu_c}\right)^2\frac{\lambda_{L,\tau}+\mu}{\lambda_{L,\tau}+2\mu}\lambda_{F,\tau}+\frac{\mu\mu_c}{\rho+\mu_c}+\frac{\rho-\mu_c}{\rho+\mu_c}\frac{\mu^2}{\lambda_{L,\tau}+2\mu}.\end{aligned}$$

Subtracting the value at zero tax from the value at tax T gives

$$W^B_{t,T}-W^B_{t,0}=\left(\frac{\mu_c}{\rho+\mu_c}\right)^2\left[\frac{\lambda_{L,T}+\mu}{\lambda_{L,T}+2\mu}\lambda_{F,T}-\frac{\lambda_{L,0}+\mu}{\lambda_{L,0}+2\mu}\lambda_{F,0}\right]+\frac{\mu_c-\rho}{\rho+\mu_c}\left[\frac{\mu^2}{\lambda_{L,0}+2\mu}-\frac{\mu^2}{\lambda_{L,T}+2\mu}\right]\geq 0.$$

Therefore, the omnichannel total-waste difference is nonnegative. □